Computer algebra independent integration tests

4-Trig-functions/4.4-Cotangent/4.4.0-a-trg-^m-b-cot-^n

Nasser M. Abbasi

May 23, 2020 Compiled on May 23, 2020 at 12:44am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [52]. This is test number [108].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (52)	% 0. (0)
Mathematica	% 100. (52)	% 0. (0)
Maple	% 71.15 (37)	% 28.85 (15)
Maxima	% 42.31 (22)	% 57.69 (30)
Fricas	% 40.38 (21)	% 59.62 (31)
Sympy	% 15.38 (8)	% 84.62 (44)
Giac	% 26.92 (14)	% 73.08 (38)

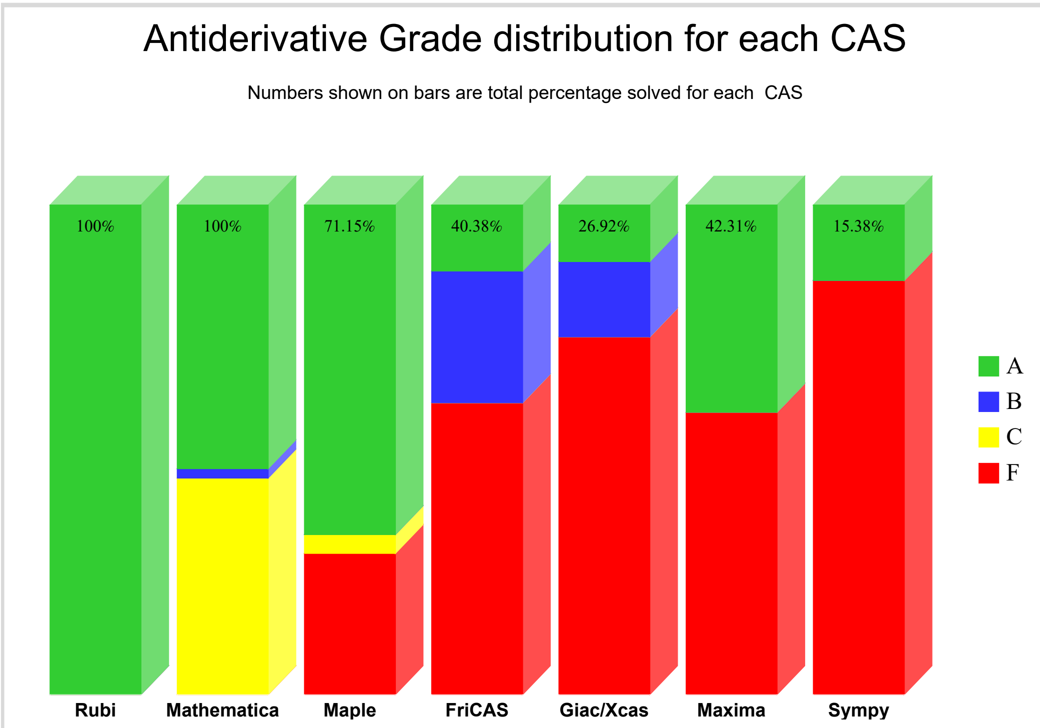
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

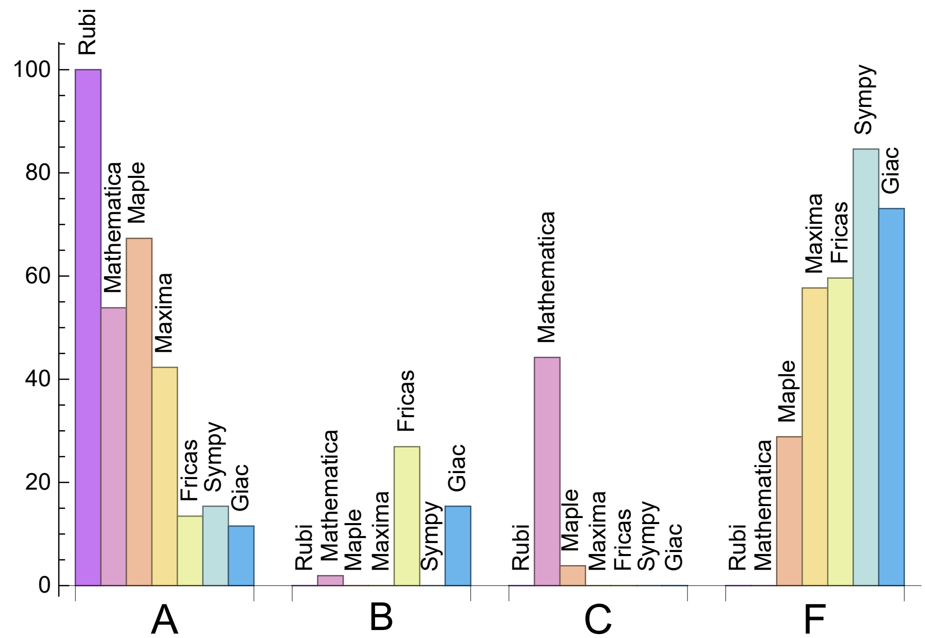
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	53.85	1.92	44.23	0.
Maple	67.31	0.	3.85	28.85
Maxima	42.31	0.	0.	57.69
Fricas	13.46	26.92	0.	59.62
Sympy	15.38	0.	0.	84.62
Giac	11.54	15.38	0.	73.08

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	104.81	1.	76.5	1.
Mathematica	0.54	105.17	1.59	40.	0.83
Maple	0.12	989.95	14.49	114.	0.91
Maxima	1.48	67.55	1.05	48.	1.11
Fricas	1.69	294.81	5.79	203.	5.08
Sympy	0.63	45.88	1.43	45.	1.3
Giac	1.21	105.43	2.87	80.5	2.74

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{39, 46, 47, 48, 50, 51, 52\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

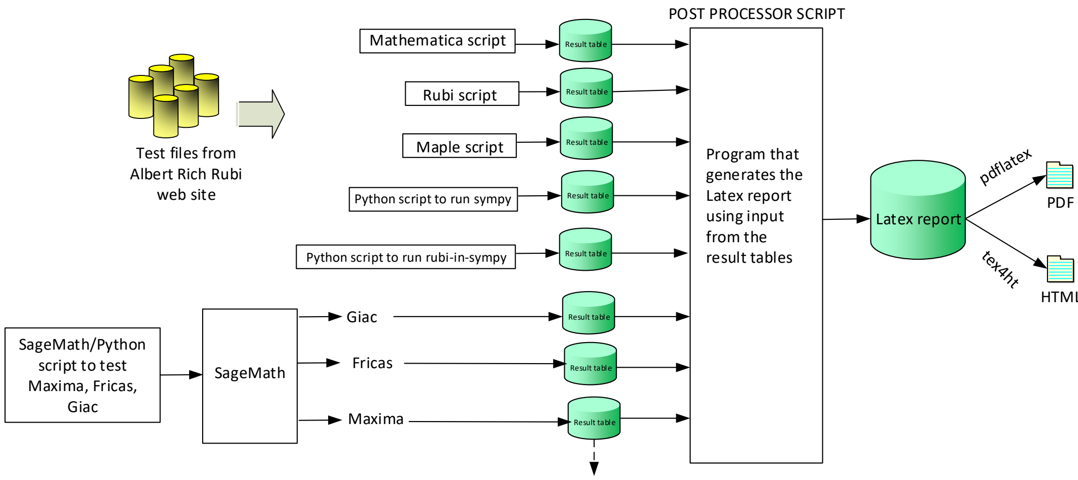
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 3, 5, 7, 9, 11, 13, 20, 23, 24, 25, 26, 27, 28, 30, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 49 }

B grade: { 48 }

C grade: { 2, 4, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 21, 22, 29, 31, 32, 39, 46, 47, 50, 51, 52 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 45 }

B grade: { }

C grade: { 43, 44 }

F grade: { 23, 24, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 52 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 19, 20, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36

B grade: { }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

2.1.5 FriCAS

A grade: { 1, 3, 25, 33, 43, 44, 45 }

B grade: { 2, 4, 5, 6, 7, 8, 19, 20, 26, 27, 28, 34, 35, 36 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 29, 30, 31, 32, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 52 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8 }

B grade: { }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

2.1.7 Giac

A grade: { 1, 25, 26, 27, 33, 34 }

B grade: { 2, 3, 4, 5, 6, 7, 8, 28 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	19	17	15	54	29	16
normalized size	1	1.	1.73	1.55	1.36	4.91	2.64	1.45
time (sec)	N/A	0.005	0.015	0.013	1.03	1.914	0.19	1.096

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	29	26	24	93	17	47
normalized size	1	1.	1.93	1.73	1.6	6.2	1.13	3.13
time (sec)	N/A	0.009	0.015	0.015	1.488	1.734	0.138	1.114

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	34	31	31	126	53	159
normalized size	1	1.	1.21	1.11	1.11	4.5	1.89	5.68
time (sec)	N/A	0.013	0.092	0.016	0.984	1.814	0.523	1.129

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	32	46	198	27	84
normalized size	1	1.	1.22	1.19	1.7	7.33	1.	3.11
time (sec)	N/A	0.017	0.013	0.017	1.458	1.232	0.227	1.127

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	46	44	51	217	66	221
normalized size	1	1.	1.1	1.05	1.21	5.17	1.57	5.26
time (sec)	N/A	0.028	0.107	0.012	1.005	1.39	0.924	1.158

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	46	59	305	39	123
normalized size	1	1.	0.73	1.02	1.31	6.78	0.87	2.73
time (sec)	N/A	0.024	0.02	0.014	1.528	1.356	0.493	1.133

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	56	57	65	319	85	281
normalized size	1	1.	0.97	0.98	1.12	5.5	1.47	4.84
time (sec)	N/A	0.03	0.296	0.014	1.055	1.347	1.596	1.199

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	33	52	73	417	51	157
normalized size	1	1.	0.58	0.91	1.28	7.32	0.89	2.75
time (sec)	N/A	0.034	0.009	0.013	1.605	1.4	0.947	1.253

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	175	200	0	0	0	0
normalized size	1	1.	0.75	0.86	0.	0.	0.	0.
time (sec)	N/A	0.192	0.508	0.057	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	40	182	0	0	0	0
normalized size	1	1.	0.19	0.86	0.	0.	0.	0.
time (sec)	N/A	0.143	0.072	0.033	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	159	176	0	0	0	0
normalized size	1	1.	0.76	0.84	0.	0.	0.	0.
time (sec)	N/A	0.14	0.191	0.03	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	40	160	0	0	0	0
normalized size	1	1.	0.21	0.83	0.	0.	0.	0.
time (sec)	N/A	0.115	0.039	0.053	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	131	166	0	0	0	0
normalized size	1	1.	0.68	0.86	0.	0.	0.	0.
time (sec)	N/A	0.112	0.086	0.064	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	38	184	0	0	0	0
normalized size	1	1.	0.18	0.87	0.	0.	0.	0.
time (sec)	N/A	0.14	0.063	0.034	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	40	184	0	0	0	0
normalized size	1	1.	0.19	0.86	0.	0.	0.	0.
time (sec)	N/A	0.143	0.069	0.033	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	40	202	0	0	0	0
normalized size	1	1.	0.17	0.86	0.	0.	0.	0.
time (sec)	N/A	0.171	0.112	0.035	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	38	214	0	0	0	0
normalized size	1	1.	0.16	0.88	0.	0.	0.	0.
time (sec)	N/A	0.47	0.03	0.092	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	40	203	0	0	0	0
normalized size	1	1.	0.18	0.9	0.	0.	0.	0.
time (sec)	N/A	0.38	0.047	0.065	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	40	114	162	560	0	0
normalized size	1	1.	0.31	0.87	1.24	4.27	0.	0.
time (sec)	N/A	0.102	0.04	0.028	1.635	1.67	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	98	114	163	1639	0	0
normalized size	1	1.	0.75	0.87	1.24	12.51	0.	0.
time (sec)	N/A	0.096	0.151	0.024	1.573	1.771	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	38	209	0	0	0	0
normalized size	1	1.	0.17	0.93	0.	0.	0.	0.
time (sec)	N/A	0.31	0.026	0.059	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	38	229	0	0	0	0
normalized size	1	1.	0.16	0.94	0.	0.	0.	0.
time (sec)	N/A	0.425	0.057	0.062	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	48	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.044	0.39	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.068	0.378	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	29	41	139	0	42
normalized size	1	1.	0.75	0.81	1.14	3.86	0.	1.17
time (sec)	N/A	0.018	0.019	0.055	1.545	1.621	0.	1.207

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	27	126	0	27
normalized size	1	1.	1.	1.38	1.69	7.88	0.	1.69
time (sec)	N/A	0.021	0.007	0.073	1.626	1.649	0.	1.244

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	28	16	128	0	26
normalized size	1	1.	1.	1.65	0.94	7.53	0.	1.53
time (sec)	N/A	0.012	0.008	0.086	1.553	1.673	0.	1.267

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	30	36	30	198	0	188
normalized size	1	1.	0.77	0.92	0.77	5.08	0.	4.82
time (sec)	N/A	0.018	0.028	0.046	1.546	1.689	0.	1.54

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	39	189	153	0	0	0
normalized size	1	1.	0.2	0.94	0.76	0.	0.	0.
time (sec)	N/A	0.097	0.055	0.062	1.562	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	122	165	127	0	0	0
normalized size	1	1.	0.69	0.94	0.72	0.	0.	0.
time (sec)	N/A	0.087	0.105	0.079	1.621	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	28	164	127	0	0	0
normalized size	1	1.	0.16	0.93	0.72	0.	0.	0.
time (sec)	N/A	0.09	0.011	0.078	1.692	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	30	185	147	0	0	0
normalized size	1	1.	0.14	0.87	0.69	0.	0.	0.
time (sec)	N/A	0.096	0.014	0.051	1.578	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	39	40	50	289	0	77
normalized size	1	1.	0.56	0.57	0.71	4.13	0.	1.1
time (sec)	N/A	0.027	0.135	0.06	1.655	2.177	0.	1.232

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	20	27	22	154	0	28
normalized size	1	1.	0.62	0.84	0.69	4.81	0.	0.88
time (sec)	N/A	0.015	0.015	0.078	1.644	2.159	0.	1.299

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	21	26	18	203	0	0
normalized size	1	1.	0.68	0.84	0.58	6.55	0.	0.
time (sec)	N/A	0.016	0.022	0.084	1.522	2.109	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	42	42	39	367	0	0
normalized size	1	1.	0.55	0.55	0.51	4.77	0.	0.
time (sec)	N/A	0.026	0.105	0.047	1.661	1.623	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.052	4.602	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	0.047	5.854	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	289	0	0	0	0	0
normalized size	1	1.	3.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	1.722	1.175	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.511	1.02	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.088	0.507	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	83	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	0.445	1.068	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	21900	0	358	0	0
normalized size	1	1.	0.96	288.16	0.	4.71	0.	0.
time (sec)	N/A	0.071	0.234	2.195	0.	1.751	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	45	10907	0	204	0	0
normalized size	1	1.	0.88	213.86	0.	4.	0.	0.
time (sec)	N/A	0.053	0.127	0.757	0.	1.732	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	26	26	0	97	0	0
normalized size	1	1.	1.04	1.04	0.	3.88	0.	0.
time (sec)	N/A	0.042	0.019	0.03	0.	1.728	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	509	0	0	0	0	0
normalized size	1	1.	9.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	3.074	1.099	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	1099	0	0	0	0	0
normalized size	1	1.	21.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	7.333	1.15	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	190	0	0	0	0	0
normalized size	1	1.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	6.513	0.551	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	69	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.131	0.499	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	264	0	0	0	0	0
normalized size	1	1.	3.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	1.051	1.	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	477	0	0	0	0	0
normalized size	1	1.	6.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	2.308	1.074	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	306	0	0	0	0	0
normalized size	1	1.	3.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	1.798	1.115	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [29] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	6	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2	A	2	2	1.	8	0.25
3	A	2	2	1.	8	0.25
4	A	3	2	1.	8	0.25
5	A	3	2	1.	8	0.25
6	A	4	2	1.	8	0.25
7	A	4	2	1.	8	0.25
8	A	5	2	1.	8	0.25
9	A	13	9	1.	12	0.75
10	A	12	9	1.	12	0.75
11	A	12	9	1.	12	0.75
12	A	11	8	1.	12	0.667
13	A	11	8	1.	12	0.667
14	A	12	9	1.	12	0.75
15	A	12	9	1.	12	0.75
16	A	13	9	1.	12	0.75
17	A	13	9	1.	12	0.75
18	A	12	8	1.	12	0.667
19	A	9	9	1.	12	0.75
20	A	9	9	1.	12	0.75
21	A	12	8	1.	12	0.667
22	A	13	9	1.	12	0.75
23	A	2	2	1.	8	0.25
24	A	2	2	1.	10	0.2
25	A	3	3	1.	10	0.3
26	A	2	2	1.	10	0.2
27	A	2	2	1.	10	0.2
28	A	3	3	1.	10	0.3
29	A	14	10	1.	10	1.
30	A	13	10	1.	10	1.
31	A	13	10	1.	10	1.
32	A	14	10	1.	10	1.
33	A	5	3	1.	10	0.3

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
34	A	3	3	1.	10	0.3
35	A	3	3	1.	10	0.3
36	A	5	3	1.	10	0.3
37	A	3	3	1.	12	0.25
38	A	3	3	1.	14	0.214
39	A	2	2	1.	21	0.095
40	A	2	2	1.	21	0.095
41	A	3	3	1.	21	0.143
42	A	3	3	1.	21	0.143
43	A	3	2	1.	19	0.105
44	A	3	2	1.	19	0.105
45	A	2	2	1.	19	0.105
46	A	2	2	1.	19	0.105
47	A	2	2	1.	19	0.105
48	A	1	1	1.	19	0.053
49	A	1	1	1.	17	0.059
50	A	1	1	1.	17	0.059
51	A	1	1	1.	19	0.053
52	A	1	1	1.	21	0.048

Chapter 3

Listing of integrals

3.1 $\int \cot(a + bx) \, dx$

Optimal. Leaf size=11

$$\frac{\log(\sin(a + bx))}{b}$$

```
[Out] Log[Sin[a + b*x]]/b
```

Rubi [A] time = 0.004614, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b}$$

Mathematica [A] time = 0.0147557, size = 19, normalized size = 1.73

$$\frac{\log(\tan(a+bx)) + \log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x],x]
```

```
[Out] (Log[Cos[a + b*x]] + Log[Tan[a + b*x]])/b
```

Maple [A] time = 0.013, size = 17, normalized size = 1.6

$$-\frac{\ln\left((\cot(bx+a))^2+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a),x)
```

```
[Out] -1/2/b*ln(cot(b*x+a)^2+1)
```

Maxima [A] time = 1.0295, size = 15, normalized size = 1.36

$$\frac{\log(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sin(b*x + a))/b
```

Fricas [A] time = 1.91427, size = 54, normalized size = 4.91

$$\frac{\log\left(-\frac{1}{2}\cos\left(2\,bx+2\,a\right)+\frac{1}{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*b*x + 2*a) + 1/2)/b
```

Sympy [A] time = 0.190214, size = 29, normalized size = 2.64

$$\begin{cases} -\frac{\log\left(\tan^{2}\left(a+bx\right)+1\right)}{2b}+\frac{\log\left(\tan\left(a+bx\right)\right)}{b} & \text{for } b\neq 0\\ x\cot\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b, Ne(b, 0))
, (x*cot(a), True))
```

Giac [A] time = 1.09626, size = 16, normalized size = 1.45

$$\frac{\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(sin(b*x + a)))/b
```

3.2 $\int \cot^2(a + bx)\, dx$

Optimal. Leaf size=15

$$-\frac{\cot(a+bx)}{b}-x$$

```
[Out] -x - Cot[a + b*x]/b
```

Rubi [A] time = 0.0087821, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-\frac{\cot(a+bx)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^2,x]
```

```
[Out] -x - Cot[a + b*x]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(a+bx)\,dx &= -\frac{\cot(a+bx)}{b}-\int 1\,dx\\ &= -x-\frac{\cot(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.0146062, size = 29, normalized size = 1.93

$$-\frac{\cot(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^2,x]
```

```
[Out] -((Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b)
```

Maple [A] time = 0.015, size = 26, normalized size = 1.7

$$\frac{1}{b}\left(-\cot(bx+a)+\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2,x)
```

```
[Out] 1/b*(-cot(b*x+a)+1/2*Pi-arccot(cot(b*x+a)))
```

Maxima [A] time = 1.48801, size = 24, normalized size = 1.6

$$-\frac{bx+a+\frac{1}{\tan(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b*x + a + 1/tan(b*x + a))/b
```

Fricas [B] time = 1.73378, size = 93, normalized size = 6.2

$$-\frac{bx\sin(2bx+2a)+\cos(2bx+2a)+1}{b\sin(2bx+2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1)/(b*sin(2*b*x + 2*a))
```

Sympy [A] time = 0.138366, size = 17, normalized size = 1.13

$$\begin{cases} -x - \frac{\cot{\left(a + bx \right)}}{b} & \text{for}\: b \neq 0 \\ x \cot^{2}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2,x)
```

```
[Out] Piecewise((-x - cot(a + b*x)/b, Ne(b, 0)), (x*cot(a)**2, True))
```

Giac [B] time = 1.11432, size = 47, normalized size = 3.13

$$-\frac{2\,bx + 2\,a + \frac{1}{\tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)} - \tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + 2*a + 1/tan(1/2*b*x + 1/2*a) - tan(1/2*b*x + 1/2*a))/b
```

3.3 $\int \cot^3(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}$$

```
[Out] -Cot[a + b*x]^2/(2*b) - Log[Sin[a + b*x]]/b
```

Rubi [A] time = 0.0126411, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$-\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^3,x]
```

```
[Out] -Cot[a + b*x]^2/(2*b) - Log[Sin[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^3(a+bx)\,dx &= -\frac{\cot^2(a+bx)}{2b}-\int \cot(a+bx)\,dx\\ &= -\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.0917749, size = 34, normalized size = 1.21

$$-\frac{\cot^2(a+bx)+2\log(\tan(a+bx))+2\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^3,x]
```

```
[Out] -(Cot[a + b*x]^2 + 2*Log[Cos[a + b*x]] + 2*Log[Tan[a + b*x]])/(2*b)
```

Maple [A] time = 0.016, size = 31, normalized size = 1.1

$$-\frac{(\cot(bx+a))^2}{2b}+\frac{\ln\left((\cot(bx+a))^2+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3,x)
```

```
[Out] -1/2*cot(b*x+a)^2/b+1/2/b*ln(cot(b*x+a)^2+1)
```

Maxima [A] time = 0.983791, size = 31, normalized size = 1.11

$$-\frac{\frac{1}{\sin(bx+a)^2}+\log\left(\sin(bx+a)^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b
```

Fricas [A] time = 1.81369, size = 126, normalized size = 4.5

$$-\frac{(\cos(2bx+2a)-1)\log\left(-\frac{1}{2}\cos(2bx+2a)+\frac{1}{2}\right)-2}{2(b\cos(2bx+2a)-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(2*b*x + 2*a) - 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2) - 2)/(b*cos(2
*b*x + 2*a) - b)
```

Sympy [A] time = 0.522513, size = 53, normalized size = 1.89

$$\begin{cases} \tilde{\infty} x & \text{for}\: a = 0 \wedge b = 0 \\ x \cot^{3}{\left (a \right)} & \text{for}\: b = 0 \\ \tilde{\infty} x & \text{for}\: a = - b x \\ \frac{\log{\left (\tan^{2}{\left (a + b x \right)} + 1 \right)}}{2 b} - \frac{\log{\left (\tan{\left (a + b x \right)} \right)}}{b} - \frac{1}{2 b \tan^{2}{\left (a + b x \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*cot(a)**3, Eq(b, 0)), (zoo*x, Eq
(a, -b*x)), (log(tan(a + b*x)**2 + 1)/(2*b) - log(tan(a + b*x))/b - 1/(2*b*
tan(a + b*x)**2), True))
```

Giac [B] time = 1.12876, size = 159, normalized size = 5.68

$$\frac{\frac{\left(\frac{4 \left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}+1\right)\left(\cos(bx+a)+1\right)}{\cos(bx+a)-1} + \frac{\cos(bx+a)-1}{\cos(bx+a)+1} - 4 \log\left(\frac{\left|-\cos(bx+a)+1\right|}{\left|\cos(bx+a)+1\right|}\right) + 8 \log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((4*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(cos(
b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 4*log(abs(-cos(b*x
+ a) + 1)/abs(cos(b*x + a) + 1)) + 8*log(abs(-(cos(b*x + a) - 1)/(cos(b*x +
 a) + 1) + 1)))/b
```

3.4 $\int \cot^4(a+bx)\,dx$

Optimal. Leaf size=27

$$-\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}+x$$

```
[Out] x + Cot[a + b*x]/b - Cot[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0173004, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^4,x]
```

```
[Out] x + Cot[a + b*x]/b - Cot[a + b*x]^3/(3*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(a+bx)\,dx &= -\frac{\cot^3(a+bx)}{3b}-\int \cot^2(a+bx)\,dx\\
&= \frac{\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\int 1\,dx\\
&= x+\frac{\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [C] time = 0.0127684, size = 33, normalized size = 1.22

$$-\frac{\cot^3(a+bx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^4,x]
```

```
[Out] -(Cot[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[a + b*x]^2])/(3*b)
```

Maple [A] time = 0.017, size = 32, normalized size = 1.2

$$\frac{1}{b}\left(-\frac{(\cot(bx+a))^3}{3}+\cot(bx+a)-\frac{\pi}{2}+\operatorname{arccot}(\cot(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3*cot(b*x+a)^3+cot(b*x+a)-1/2*Pi+arccot(cot(b*x+a)))
```

Maxima [A] time = 1.45773, size = 46, normalized size = 1.7

$$\frac{3\,bx+3\,a+\frac{3\,\tan(bx+a)^2-1}{\tan(bx+a)^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*x + 3*a + (3*tan(b*x + a)^2 - 1)/tan(b*x + a)^3)/b
```

Fricas [B] time = 1.23237, size = 198, normalized size = 7.33

$$\frac{4\,\cos(2\,bx+2\,a)^2+3\,(bx\cos(2\,bx+2\,a)-bx)\sin(2\,bx+2\,a)+2\,\cos(2\,bx+2\,a)-2}{3\,(b\cos(2\,bx+2\,a)-b)\sin(2\,bx+2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*b*x + 2*a)^2 + 3*(b*x*cos(2*b*x + 2*a) - b*x)*sin(2*b*x + 2*a)
 + 2*cos(2*b*x + 2*a) - 2)/((b*cos(2*b*x + 2*a) - b)*sin(2*b*x + 2*a))
```

Sympy [A] time = 0.227265, size = 27, normalized size = 1.

$$\begin{cases} x - \frac{\cot^3(a+bx)}{3b} + \frac{\cot(a+bx)}{b} & \text{for } b \neq 0 \\ x\cot^4(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**4,x)
```

```
[Out] Piecewise((x - cot(a + b*x)**3/(3*b) + cot(a + b*x)/b, Ne(b, 0)), (x*cot(a)
**4, True))
```

Giac [B] time = 1.12685, size = 84, normalized size = 3.11

$$\frac{\tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)^3 + 24\,bx + 24\,a + \frac{15\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2 - 1}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^3} - 15\,\tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/24*(tan(1/2*b*x + 1/2*a)^3 + 24*b*x + 24*a + (15*tan(1/2*b*x + 1/2*a)^2 -
 1)/tan(1/2*b*x + 1/2*a)^3 - 15*tan(1/2*b*x + 1/2*a))/b
```

3.5 $\int \cot^5(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\cot^4(a+bx)}{4b}+\frac{\cot^2(a+bx)}{2b}+\frac{\log(\sin(a+bx))}{b}$$

```
[Out] Cot[a + b*x]^2/(2*b) - Cot[a + b*x]^4/(4*b) + Log[Sin[a + b*x]]/b
```

Rubi [A] time = 0.027822, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$-\frac{\cot^4(a+bx)}{4b}+\frac{\cot^2(a+bx)}{2b}+\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^5,x]
```

```
[Out] Cot[a + b*x]^2/(2*b) - Cot[a + b*x]^4/(4*b) + Log[Sin[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^5(a+bx)\,dx &= -\frac{\cot^4(a+bx)}{4b}-\int \cot^3(a+bx)\,dx\\
&= \frac{\cot^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}+\int \cot(a+bx)\,dx\\
&= \frac{\cot^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}+\frac{\log(\sin(a+bx))}{b}
\end{aligned}$$

Mathematica [A] time = 0.106946, size = 46, normalized size = 1.1

$$\frac{-\cot^4(a+bx)+2\cot^2(a+bx)+4\log(\tan(a+bx))+4\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^5,x]
```

```
[Out] (2*Cot[a + b*x]^2 - Cot[a + b*x]^4 + 4*Log[Cos[a + b*x]] + 4*Log[Tan[a + b*
x]])/(4*b)
```

Maple [A] time = 0.012, size = 44, normalized size = 1.1

$$-\frac{(\cot(bx+a))^4}{4b}+\frac{(\cot(bx+a))^2}{2b}-\frac{\ln\left((\cot(bx+a))^2+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^5,x)
```

```
[Out] -1/4*cot(b*x+a)^4/b+1/2*cot(b*x+a)^2/b-1/2/b*ln(cot(b*x+a)^2+1)
```

Maxima [A] time = 1.00492, size = 51, normalized size = 1.21

$$\frac{\frac{4\sin(bx+a)^2-1}{\sin(bx+a)^4}+2\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*((4*sin(b*x + a)^2 - 1)/sin(b*x + a)^4 + 2*log(sin(b*x + a)^2))/b
```

Fricas [B] time = 1.39003, size = 217, normalized size = 5.17

$$\frac{\left(\cos(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(-\frac{1}{2}\cos(2bx+2a)+\frac{1}{2}\right)-4\cos(2bx+2a)+2}{2\left(b\cos(2bx+2a)^2-2b\cos(2bx+2a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/2*((cos(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(-1/2*cos(2*b*x + 2*a
) + 1/2) - 4*cos(2*b*x + 2*a) + 2)/(b*cos(2*b*x + 2*a)^2 - 2*b*cos(2*b*x +
2*a) + b)
```

Sympy [A] time = 0.923948, size = 66, normalized size = 1.57

$$\begin{cases} \tilde{\infty} x & \text{for}\: a = 0 \wedge b = 0 \\ x \cot^{5}{\left(a \right)} & \text{for}\: b = 0 \\ \tilde{\infty} x & \text{for}\: a = - b x \\ - \frac{\log{\left(\tan^{2}{\left(a + b x \right)} + 1 \right)}}{2 b} + \frac{\log{\left(\tan{\left(a + b x \right)} \right)}}{b} + \frac{1}{2 b \tan^{2}{\left(a + b x \right)}} - \frac{1}{4 b \tan^{4}{\left(a + b x \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**5,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*cot(a)**5, Eq(b, 0)), (zoo*x, Eq
(a, -b*x)), (-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b + 1/(2*b
*tan(a + b*x)**2) - 1/(4*b*tan(a + b*x)**4), True))
```

Giac [B] time = 1.15783, size = 221, normalized size = 5.26

$$-\frac{\frac{\left(\frac{12 (\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{48 (\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} + \frac{12 (\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 32 \log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) + 64 \log\left(\left|-\frac{\cos(bx+a)}{\cos(bx+a)}\right.\right.}{64\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 48*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 12*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 - 32*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 64*log(abs(-(
cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.6 $\int \cot^6(a+bx)\,dx$

Optimal. Leaf size=45

$$-\frac{\cot^5(a+bx)}{5b}+\frac{\cot^3(a+bx)}{3b}-\frac{\cot(a+bx)}{b}-x$$

```
[Out] -x - Cot[a + b*x]/b + Cot[a + b*x]^3/(3*b) - Cot[a + b*x]^5/(5*b)
```

Rubi [A] time = 0.0244749, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-\frac{\cot^5(a+bx)}{5b}+\frac{\cot^3(a+bx)}{3b}-\frac{\cot(a+bx)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^6,x]
```

```
[Out] -x - Cot[a + b*x]/b + Cot[a + b*x]^3/(3*b) - Cot[a + b*x]^5/(5*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(a+bx)\,dx &= -\frac{\cot^5(a+bx)}{5b} - \int \cot^4(a+bx)\,dx \\
&= \frac{\cot^3(a+bx)}{3b} - \frac{\cot^5(a+bx)}{5b} + \int \cot^2(a+bx)\,dx \\
&= -\frac{\cot(a+bx)}{b} + \frac{\cot^3(a+bx)}{3b} - \frac{\cot^5(a+bx)}{5b} - \int 1\,dx \\
&= -x - \frac{\cot(a+bx)}{b} + \frac{\cot^3(a+bx)}{3b} - \frac{\cot^5(a+bx)}{5b}
\end{aligned}$$

Mathematica [C] time = 0.0202553, size = 33, normalized size = 0.73

$$-\frac{\cot^5(a+bx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^6,x]
```

```
[Out] -(Cot[a + b*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[a + b*x]^2])/(5*b)
```

Maple [A] time = 0.014, size = 46, normalized size = 1.

$$\frac{1}{b}\left(-\frac{(\cot(bx+a))^5}{5} + \frac{(\cot(bx+a))^3}{3} - \cot(bx+a) + \frac{\pi}{2} - \operatorname{arccot}(\cot(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^6,x)
```

```
[Out] 1/b*(-1/5*cot(b*x+a)^5+1/3*cot(b*x+a)^3-cot(b*x+a)+1/2*Pi-arccot(cot(b*x+a)
))
```

Maxima [A] time = 1.52765, size = 59, normalized size = 1.31

$$-\frac{15\,bx + 15\,a + \frac{15\,\tan(bx+a)^4 - 5\,\tan(bx+a)^2 + 3}{\tan(bx+a)^5}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b*x + 15*a + (15*tan(b*x + a)^4 - 5*tan(b*x + a)^2 + 3)/tan(b*x +
 a)^5)/b
```

Fricas [B] time = 1.35594, size = 305, normalized size = 6.78

$$-\frac{23 \cos\left(2 bx+2 a\right)^{3}-\cos\left(2 bx+2 a\right)^{2}+15 \left(bx \cos\left(2 bx+2 a\right)^{2}-2 bx \cos\left(2 bx+2 a\right)+bx\right) \sin\left(2 bx+2 a\right)-11 \cos(2}{15 \left(b \cos\left(2 bx+2 a\right)^{2}-2 b \cos\left(2 bx+2 a\right)+b\right) \sin\left(2 bx+2 a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(23*cos(2*b*x + 2*a)^3 - cos(2*b*x + 2*a)^2 + 15*(b*x*cos(2*b*x + 2*a
)^2 - 2*b*x*cos(2*b*x + 2*a) + b*x)*sin(2*b*x + 2*a) - 11*cos(2*b*x + 2*a)
+ 13)/((b*cos(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2*a) + b)*sin(2*b*x + 2*a))
```

Sympy [A] time = 0.492814, size = 39, normalized size = 0.87

$$\begin{cases} -x - \frac{\cot^{5}{\left(a+bx \right)}}{5b} + \frac{\cot^{3}{\left(a+bx \right)}}{3b} - \frac{\cot{\left(a+bx \right)}}{b} & \text{for}\: b \neq 0 \\ x \cot^{6}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**6,x)
```

```
[Out] Piecewise((-x - cot(a + b*x)**5/(5*b) + cot(a + b*x)**3/(3*b) - cot(a + b*x
)/b, Ne(b, 0)), (x*cot(a)**6, True))
```

Giac [B] time = 1.13303, size = 123, normalized size = 2.73

$$\frac{3 \tan\left(\frac{1}{2} bx+\frac{1}{2} a\right)^{5}-35 \tan\left(\frac{1}{2} bx+\frac{1}{2} a\right)^{3}-480 bx-480 a-\frac{330 \tan\left(\frac{1}{2} bx+\frac{1}{2} a\right)^{4}-35 \tan\left(\frac{1}{2} bx+\frac{1}{2} a\right)^{2}+3}{\tan\left(\frac{1}{2} bx+\frac{1}{2} a\right)^{5}}+330 \tan\left(\frac{1}{2} bx+\frac{1}{2} a\right)}{480 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 1/480*(3*tan(1/2*b*x + 1/2*a)^5 - 35*tan(1/2*b*x + 1/2*a)^3 - 480*b*x - 480
*a - (330*tan(1/2*b*x + 1/2*a)^4 - 35*tan(1/2*b*x + 1/2*a)^2 + 3)/tan(1/2*b
*x + 1/2*a)^5 + 330*tan(1/2*b*x + 1/2*a))/b
```

3.7 $\int \cot^7(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\cot^6(a+bx)}{6b}+\frac{\cot^4(a+bx)}{4b}-\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}$$

```
[Out] -Cot[a + b*x]^2/(2*b) + Cot[a + b*x]^4/(4*b) - Cot[a + b*x]^6/(6*b) - Log[S
in[a + b*x]]/b
```

Rubi [A] time = 0.0297959, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 3475}

$$-\frac{\cot^6(a+bx)}{6b}+\frac{\cot^4(a+bx)}{4b}-\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^7,x]
```

```
[Out] -Cot[a + b*x]^2/(2*b) + Cot[a + b*x]^4/(4*b) - Cot[a + b*x]^6/(6*b) - Log[S
in[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^7(a+bx)\,dx &= -\frac{\cot^6(a+bx)}{6b} - \int \cot^5(a+bx)\,dx \\ &= \frac{\cot^4(a+bx)}{4b} - \frac{\cot^6(a+bx)}{6b} + \int \cot^3(a+bx)\,dx \\ &= -\frac{\cot^2(a+bx)}{2b} + \frac{\cot^4(a+bx)}{4b} - \frac{\cot^6(a+bx)}{6b} - \int \cot(a+bx)\,dx \\ &= -\frac{\cot^2(a+bx)}{2b} + \frac{\cot^4(a+bx)}{4b} - \frac{\cot^6(a+bx)}{6b} - \frac{\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.295689, size = 56, normalized size = 0.97

$$-\frac{2\cot^6(a+bx) - 3\cot^4(a+bx) + 6\cot^2(a+bx) + 12\log(\tan(a+bx)) + 12\log(\cos(a+bx))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^7,x]
```

```
[Out] -(6*Cot[a + b*x]^2 - 3*Cot[a + b*x]^4 + 2*Cot[a + b*x]^6 + 12*Log[Cos[a + b
*x]] + 12*Log[Tan[a + b*x]])/(12*b)
```

Maple [A] time = 0.014, size = 57, normalized size = 1.

$$-\frac{(\cot(bx+a))^6}{6b} + \frac{(\cot(bx+a))^4}{4b} - \frac{(\cot(bx+a))^2}{2b} + \frac{\ln\left((\cot(bx+a))^2+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^7,x)
```

```
[Out] -1/6*cot(b*x+a)^6/b+1/4*cot(b*x+a)^4/b-1/2*cot(b*x+a)^2/b+1/2/b*ln(cot(b*x+
a)^2+1)
```

Maxima [A] time = 1.05457, size = 65, normalized size = 1.12

$$-\frac{\frac{18\sin(bx+a)^4 - 9\sin(bx+a)^2 + 2}{\sin(bx+a)^6} + 6\log\left(\sin(bx+a)^2\right)}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/12*((18*sin(b*x + a)^4 - 9*sin(b*x + a)^2 + 2)/sin(b*x + a)^6 + 6*log(si
n(b*x + a)^2))/b
```

Fricas [B] time = 1.34652, size = 319, normalized size = 5.5

$$\frac{18\,\cos\left(2\,bx+2\,a\right)^2-3\left(\cos\left(2\,bx+2\,a\right)^3-3\,\cos\left(2\,bx+2\,a\right)^2+3\,\cos\left(2\,bx+2\,a\right)-1\right)\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)+\frac{1}{2}\right)-18}{6\left(b\cos\left(2\,bx+2\,a\right)^3-3\,b\cos\left(2\,bx+2\,a\right)^2+3\,b\cos\left(2\,bx+2\,a\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/6*(18*cos(2*b*x + 2*a)^2 - 3*(cos(2*b*x + 2*a)^3 - 3*cos(2*b*x + 2*a)^2 +
 3*cos(2*b*x + 2*a) - 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2) - 18*cos(2*b*x +
2*a) + 8)/(b*cos(2*b*x + 2*a)^3 - 3*b*cos(2*b*x + 2*a)^2 + 3*b*cos(2*b*x +
2*a) - b)
```

Sympy [A] time = 1.59554, size = 85, normalized size = 1.47

$$\begin{cases} \tilde{\infty}x & \text{for}\ \left(a=0 \vee a=-bx\right)\wedge\left(a=-bx \vee b=0\right) \\ x\cot^7\left(a\right) & \text{for}\ b=0 \\ \frac{\log\left(\tan^2\left(a+bx\right)+1\right)}{2b}-\frac{\log\left(\tan\left(a+bx\right)\right)}{b}-\frac{1}{2b\tan^2\left(a+bx\right)}+\frac{1}{4b\tan^4\left(a+bx\right)}-\frac{1}{6b\tan^6\left(a+bx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**7,x)
```

```
[Out] Piecewise((zoo*x, (Eq(a, 0) | Eq(a, -b*x)) & (Eq(b, 0) | Eq(a, -b*x))), (x*
cot(a)**7, Eq(b, 0)), (log(tan(a + b*x)**2 + 1)/(2*b) - log(tan(a + b*x))/b
 - 1/(2*b*tan(a + b*x)**2) + 1/(4*b*tan(a + b*x)**4) - 1/(6*b*tan(a + b*x)*
*6), True))
```

<u>Giac</u> [B] time = 1.19852, size = 281, normalized size = 4.84

$$\frac{\frac{\left(\frac{12 (\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{87 (\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{352 (\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+1\right)(\cos(bx+a)+1)^3}{(\cos(bx+a)-1)^3}+\frac{87 (\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{12 (\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-192 \log\left(\frac{|-\cdots}{|\cdots}\right. \cdots}{384 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/384*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 87*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 352*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 1)*(
cos(b*x + a) + 1)^3/(cos(b*x + a) - 1)^3 + 87*(cos(b*x + a) - 1)/(cos(b*x +
 a) + 1) + 12*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + (cos(b*x + a) - 1
)^3/(cos(b*x + a) + 1)^3 - 192*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a)
+ 1)) + 384*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.8 $\int \cot^8(a+bx)\,dx$

Optimal. Leaf size=57

$$-\frac{\cot^7(a+bx)}{7b}+\frac{\cot^5(a+bx)}{5b}-\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}+x$$

```
[Out] x + Cot[a + b*x]/b - Cot[a + b*x]^3/(3*b) + Cot[a + b*x]^5/(5*b) - Cot[a +
b*x]^7/(7*b)
```

Rubi [A] time = 0.0335139, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-\frac{\cot^7(a+bx)}{7b}+\frac{\cot^5(a+bx)}{5b}-\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^8,x]
```

```
[Out] x + Cot[a + b*x]/b - Cot[a + b*x]^3/(3*b) + Cot[a + b*x]^5/(5*b) - Cot[a +
b*x]^7/(7*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^8(a+bx)\,dx &= -\frac{\cot^7(a+bx)}{7b} - \int \cot^6(a+bx)\,dx \\ &= \frac{\cot^5(a+bx)}{5b} - \frac{\cot^7(a+bx)}{7b} + \int \cot^4(a+bx)\,dx \\ &= -\frac{\cot^3(a+bx)}{3b} + \frac{\cot^5(a+bx)}{5b} - \frac{\cot^7(a+bx)}{7b} - \int \cot^2(a+bx)\,dx \\ &= \frac{\cot(a+bx)}{b} - \frac{\cot^3(a+bx)}{3b} + \frac{\cot^5(a+bx)}{5b} - \frac{\cot^7(a+bx)}{7b} + \int 1\,dx \\ &= x + \frac{\cot(a+bx)}{b} - \frac{\cot^3(a+bx)}{3b} + \frac{\cot^5(a+bx)}{5b} - \frac{\cot^7(a+bx)}{7b}\end{aligned}$$

Mathematica [C] time = 0.0091447, size = 33, normalized size = 0.58

$$-\frac{\cot^7(a+bx)\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(a+bx)\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^8,x]
```

```
[Out] -(Cot[a + b*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[a + b*x]^2])/(7*b)
```

Maple [A] time = 0.013, size = 52, normalized size = 0.9

$$\frac{1}{b}\left(-\frac{(\cot(bx+a))^7}{7}+\frac{(\cot(bx+a))^5}{5}-\frac{(\cot(bx+a))^3}{3}+\cot(bx+a)-\frac{\pi}{2}+\operatorname{arccot}(\cot(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^8,x)
```

```
[Out] 1/b*(-1/7*cot(b*x+a)^7+1/5*cot(b*x+a)^5-1/3*cot(b*x+a)^3+cot(b*x+a)-1/2*Pi+
arccot(cot(b*x+a)))
```

Maxima [A] time = 1.60531, size = 73, normalized size = 1.28

$$\frac{105\,bx + 105\,a + \frac{105\,\tan\left(bx+a\right)^{6} - 35\,\tan\left(bx+a\right)^{4} + 21\,\tan\left(bx+a\right)^{2} - 15}{\tan\left(bx+a\right)^{7}}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] 1/105*(105*b*x + 105*a + (105*tan(b*x + a)^6 - 35*tan(b*x + a)^4 + 21*tan(b
*x + a)^2 - 15)/tan(b*x + a)^7)/b
```

Fricas [B] time = 1.40016, size = 417, normalized size = 7.32

$$\frac{176\,\cos\left(2\,bx + 2\,a\right)^{4} - 108\,\cos\left(2\,bx + 2\,a\right)^{3} + 20\,\cos\left(2\,bx + 2\,a\right)^{2} + 105\left(bx\cos\left(2\,bx + 2\,a\right)^{3} - 3\,bx\cos\left(2\,bx + 2\,a\right)^{2} + 3}{105\left(b\cos\left(2\,bx + 2\,a\right)^{3} - 3\,b\cos\left(2\,bx + 2\,a\right)^{2} + 3\,b\cos\left(2\,bx + 2\,a\right) - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(176*cos(2*b*x + 2*a)^4 - 108*cos(2*b*x + 2*a)^3 + 20*cos(2*b*x + 2*a
)^2 + 105*(b*x*cos(2*b*x + 2*a)^3 - 3*b*x*cos(2*b*x + 2*a)^2 + 3*b*x*cos(2*
b*x + 2*a) - b*x)*sin(2*b*x + 2*a) + 228*cos(2*b*x + 2*a) - 76)/((b*cos(2*b
*x + 2*a)^3 - 3*b*cos(2*b*x + 2*a)^2 + 3*b*cos(2*b*x + 2*a) - b)*sin(2*b*x
+ 2*a))
```

Sympy [A] time = 0.947461, size = 51, normalized size = 0.89

$$\begin{cases} x - \frac{\cot^{7}{\left(a + bx \right)}}{7b} + \frac{\cot^{5}{\left(a + bx \right)}}{5b} - \frac{\cot^{3}{\left(a + bx \right)}}{3b} + \frac{\cot{\left(a + bx \right)}}{b} & \text{for}\: b \neq 0 \\ x \cot^{8}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**8,x)
```

```
[Out] Piecewise((x - cot(a + b*x)**7/(7*b) + cot(a + b*x)**5/(5*b) - cot(a + b*x)
**3/(3*b) + cot(a + b*x)/b, Ne(b, 0)), (x*cot(a)**8, True))
```

Giac [B] time = 1.25256, size = 157, normalized size = 2.75

$$\frac{15\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{7}-189\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{5}+1295\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}+13440\,bx+13440\,a+\frac{9765\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{6}-1295\,\mathrm{t}}{\mathrm{ta}}}{13440\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 1/13440*(15*tan(1/2*b*x + 1/2*a)^7 - 189*tan(1/2*b*x + 1/2*a)^5 + 1295*tan(
1/2*b*x + 1/2*a)^3 + 13440*b*x + 13440*a + (9765*tan(1/2*b*x + 1/2*a)^6 - 1
295*tan(1/2*b*x + 1/2*a)^4 + 189*tan(1/2*b*x + 1/2*a)^2 - 15)/tan(1/2*b*x +
 1/2*a)^7 - 9765*tan(1/2*b*x + 1/2*a))/b
```

3.9 $\int (c\cot(a+bx))^{7/2}\,dx$

Optimal. Leaf size=232

$$\frac{2c^3\sqrt{c\cot(a+bx)}}{b}+\frac{c^{7/2}\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}-\frac{c^{7/2}\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}\right.}{2\sqrt{2}b}$$

```
[Out] (c^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b) -
(c^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b) +
(2*c^3*Sqrt[c*Cot[a + b*x]])/b - (2*c*(c*Cot[a + b*x])^(5/2))/(5*b) + (c^(7
/2)*Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]])/(2*
Sqrt[2]*b) - (c^(7/2)*Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqrt[c*C
ot[a + b*x]]])/(2*Sqrt[2]*b)
```

Rubi [A] time = 0.191742, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2c^3\sqrt{c\cot(a+bx)}}{b}+\frac{c^{7/2}\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}-\frac{c^{7/2}\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}\right.}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(7/2),x]
```


```
[Out] (c^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b) -
(c^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b) +
(2*c^3*Sqrt[c*Cot[a + b*x]])/b - (2*c*(c*Cot[a + b*x])^(5/2))/(5*b) + (c^(7
/2)*Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]])/(2*
Sqrt[2]*b) - (c^(7/2)*Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqrt[c*C
ot[a + b*x]]])/(2*Sqrt[2]*b)
```


Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (c\cot(a+bx))^{7/2}\,dx = -\frac{2c(c\cot(a+bx))^{5/2}}{5b} - c^2\int (c\cot(a+bx))^{3/2}\,dx$$

$$= \frac{2c^3\sqrt{c\cot(a+bx)}}{b} - \frac{2c(c\cot(a+bx))^{5/2}}{5b} + c^4\int \frac{1}{\sqrt{c\cot(a+bx)}}\,dx$$

$$= \frac{2c^3\sqrt{c\cot(a+bx)}}{b} - \frac{2c(c\cot(a+bx))^{5/2}}{5b} - \frac{c^5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(c^2+x^2)}\,dx, x, c\cot(a+bx)\right)}{b}$$

$$= \frac{2c^3\sqrt{c\cot(a+bx)}}{b} - \frac{2c(c\cot(a+bx))^{5/2}}{5b} - \frac{(2c^5)\,\mathrm{Subst}\left(\int \frac{1}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b}$$

$$= \frac{2c^3\sqrt{c\cot(a+bx)}}{b} - \frac{2c(c\cot(a+bx))^{5/2}}{5b} - \frac{c^4\,\mathrm{Subst}\left(\int \frac{c-x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b} - \frac{c^4\,\mathrm{Subst}}{}$$

$$= \frac{2c^3\sqrt{c\cot(a+bx)}}{b} - \frac{2c(c\cot(a+bx))^{5/2}}{5b} + \frac{c^{7/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}+2x}{-c-\sqrt{2}\sqrt{c}x-x^2}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}b} + \cdots$$

$$= \frac{2c^3\sqrt{c\cot(a+bx)}}{b} - \frac{2c(c\cot(a+bx))^{5/2}}{5b} + \frac{c^{7/2}\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}b}$$

$$= \frac{c^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}b} - \frac{c^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}b} + \frac{2c^3\sqrt{c\cot(a+bx)}}{b} - \frac{2c(c\cot(}{5}$$

Mathematica [A] time = 0.507568, size = 175, normalized size = 0.75

$$\frac{c^3\sqrt{c\cot(a+bx)}\left(-8\cot^{\frac{5}{2}}(a+bx)+40\sqrt{\cot(a+bx)}+5\sqrt{2}\log\left(\cot(a+bx)-\sqrt{2}\sqrt{\cot(a+bx)}+1\right)-5\sqrt{2}\log\left(\cot(a+\right.\right.}{20b\sqrt{\cot(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(7/2),x]
```

```
[Out] (c^3*Sqrt[c*Cot[a + b*x]]*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*x]]
] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[a + b*x]]] + 40*Sqrt[Cot[a + b*x
]] - 8*Cot[a + b*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[a + b*x]] +
Cot[a + b*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[a + b*x]] + Cot[a + b*x]
]))/(20*b*Sqrt[Cot[a + b*x]])
```

Maple [A] time = 0.057, size = 200, normalized size = 0.9

$$-\frac{2c}{5b}(c\cot(bx+a))^{\frac{5}{2}}+2\frac{c^3\sqrt{c\cot(bx+a)}}{b}-\frac{c^3\sqrt{2}}{2b}\sqrt[4]{c^2}\arctan\left(\sqrt{2}\sqrt{c\cot(bx+a)}\frac{1}{\sqrt[4]{c^2}}+1\right)+\frac{c^3\sqrt{2}}{2b}\sqrt[4]{c^2}\arctan\left(-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cot(b*x+a))^(7/2),x)
```

```
[Out] -2/5*c*(c*cot(b*x+a))^(5/2)/b+2*c^3*(c*cot(b*x+a))^(1/2)/b-1/2/b*c^3*(c^2)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)+1)+1/2/b*c^3*
(c^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)+1)-1/4
/b*c^3*(c^2)^(1/4)*2^(1/2)*ln((c*cot(b*x+a)+(c^2)^(1/4)*(c*cot(b*x+a))^(1/2
)*2^(1/2)+(c^2)^(1/2))/(c*cot(b*x+a)-(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)*2^(1/
2)+(c^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(a+bx))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))**(7/2),x)
```

```
[Out] Integral((c*cot(a + b*x))**(7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(bx+a))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(7/2), x)
```

3.10 $\int (c\cot(a+bx))^{5/2}\,dx$

Optimal. Leaf size=212

$$\frac{c^{5/2}\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}-\frac{c^{5/2}\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}-\frac{c^{5/2}\tan^{-1}\left(\right.}{}$$

```
[Out] -((c^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b))
 + (c^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b)
 - (2*c*(c*Cot[a + b*x])^(3/2))/(3*b) + (c^(5/2)*Log[Sqrt[c] + Sqrt[c]*Cot[
a + b*x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]])/(2*Sqrt[2]*b) - (c^(5/2)*Log[Sqrt
[c] + Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x]]])/(2*Sqrt[2]*b)
```

Rubi [A] time = 0.142843, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/2}\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}-\frac{c^{5/2}\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}-\frac{c^{5/2}\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(5/2),x]
```


```
[Out] -((c^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b))
 + (c^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b)
 - (2*c*(c*Cot[a + b*x])^(3/2))/(3*b) + (c^(5/2)*Log[Sqrt[c] + Sqrt[c]*Cot[
a + b*x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]])/(2*Sqrt[2]*b) - (c^(5/2)*Log[Sqrt
[c] + Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x]]])/(2*Sqrt[2]*b)
```


Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (c\cot(a+bx))^{5/2}\,dx = -\frac{2c(c\cot(a+bx))^{3/2}}{3b} - c^2\int \sqrt{c\cot(a+bx)}\,dx$$
$$= -\frac{2c(c\cot(a+bx))^{3/2}}{3b} + \frac{c^3\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{c^2+x^2}\,dx, x, c\cot(a+bx)\right)}{b}$$
$$= -\frac{2c(c\cot(a+bx))^{3/2}}{3b} + \frac{(2c^3)\,\mathrm{Subst}\left(\int \frac{x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b}$$
$$= -\frac{2c(c\cot(a+bx))^{3/2}}{3b} - \frac{c^3\,\mathrm{Subst}\left(\int \frac{c-x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b} + \frac{c^3\,\mathrm{Subst}\left(\int \frac{c+x^2}{c^2+x^4}\,dx, x, \sqrt{c\right.}{b}$$
$$= -\frac{2c(c\cot(a+bx))^{3/2}}{3b} + \frac{c^{5/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}+2x}{-c-\sqrt{2}\sqrt{c}x-x^2}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}b} + \frac{c^{5/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}}{-c+\sqrt{}}\right.}{}$$
$$= -\frac{2c(c\cot(a+bx))^{3/2}}{3b} + \frac{c^{5/2}\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}b} - \frac{c^{5/2}\log\left(\sqrt{c}+\right.}{}$$
$$= -\frac{c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}b} + \frac{c^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}b} - \frac{2c(c\cot(a+bx))^{3/2}}{3b} + \frac{c^{5/2}\,\mathrm{l}}{}$$

Mathematica [C] time = 0.0720708, size = 40, normalized size = 0.19

$$\frac{2c(c\cot(a+bx))^{3/2}\left(\mathrm{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(a+bx)\right)-1\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(5/2),x]
```

```
[Out] (2*c*(c*Cot[a + b*x])^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b
*x]^2]))/(3*b)
```

Maple [A] time = 0.033, size = 182, normalized size = 0.9

$$-\frac{2c}{3b}(c\cot(bx+a))^{\frac{3}{2}}+\frac{c^3\sqrt{2}}{4b}\ln\left(\left(c\cot(bx+a)-\sqrt[4]{c^2}\sqrt{c\cot(bx+a)}\sqrt{2}+\sqrt{c^2}\right)\left(c\cot(bx+a)+\sqrt[4]{c^2}\sqrt{c\cot(bx+a)}\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cot(b*x+a))^(5/2),x)
```

```
[Out] -2/3*c*(c*cot(b*x+a))^(3/2)/b+1/4/b*c^3/(c^2)^(1/4)*2^(1/2)*ln((c*cot(b*x+a
)-(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)*2^(1/2)+(c^2)^(1/2))/(c*cot(b*x+a)+(c^2)
^(1/4)*(c*cot(b*x+a))^(1/2)*2^(1/2)+(c^2)^(1/2)))+1/2/b*c^3/(c^2)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)+1)-1/2/b*c^3/(c^2)^(1/
4)*2^(1/2)*arctan(-2^(1/2)/(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(a+bx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))**(5/2),x)
```

```
[Out] Integral((c*cot(a + b*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(bx+a))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(5/2), x)
```

3.11 $\int (c\cot(a+bx))^{3/2}\,dx$

Optimal. Leaf size=210

$$-\frac{c^{3/2}\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}+\frac{c^{3/2}\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}-\frac{c^{3/2}\tan^{-1}\left(\right.}{}$$

```
[Out] -((c^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b))
 + (c^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b)
 - (2*c*Sqrt[c*Cot[a + b*x]])/b - (c^(3/2)*Log[Sqrt[c] + Sqrt[c]*Cot[a + b*
x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]])/(2*Sqrt[2]*b) + (c^(3/2)*Log[Sqrt[c] +
Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x]]])/(2*Sqrt[2]*b)
```

Rubi [A] time = 0.139562, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c^{3/2}\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}+\frac{c^{3/2}\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}-\frac{c^{3/2}\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(3/2),x]
```

```
[Out] -((c^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b))
 + (c^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b)
 - (2*c*Sqrt[c*Cot[a + b*x]])/b - (c^(3/2)*Log[Sqrt[c] + Sqrt[c]*Cot[a + b*
x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]])/(2*Sqrt[2]*b) + (c^(3/2)*Log[Sqrt[c] +
Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x]]])/(2*Sqrt[2]*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (c\cot(a+bx))^{3/2}\,dx = -\frac{2c\sqrt{c\cot(a+bx)}}{b} - c^2\int \frac{1}{\sqrt{c\cot(a+bx)}}\,dx$$

$$= -\frac{2c\sqrt{c\cot(a+bx)}}{b} + \frac{c^3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(c^2+x^2)}\,dx, x, c\cot(a+bx)\right)}{b}$$

$$= -\frac{2c\sqrt{c\cot(a+bx)}}{b} + \frac{(2c^3)\,\mathrm{Subst}\left(\int \frac{1}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b}$$

$$= -\frac{2c\sqrt{c\cot(a+bx)}}{b} + \frac{c^2\,\mathrm{Subst}\left(\int \frac{c-x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b} + \frac{c^2\,\mathrm{Subst}\left(\int \frac{c+x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot}\right.}{b}$$

$$= -\frac{2c\sqrt{c\cot(a+bx)}}{b} - \frac{c^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}+2x}{-c-\sqrt{2}\sqrt{c}x-x^2}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}b} - \frac{c^{3/2}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}-2}{-c+\sqrt{2}\sqrt{c}x}\right.}{2}$$

$$= -\frac{2c\sqrt{c\cot(a+bx)}}{b} - \frac{c^{3/2}\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}b} + \frac{c^{3/2}\log\left(\sqrt{c}+\sqrt{c}\,\mathrm{c}\right.}{}$$

$$= -\frac{c^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}b} + \frac{c^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}b} - \frac{2c\sqrt{c\cot(a+bx)}}{b} - \frac{c^{3/2}\log\left(\right.}{}$$

Mathematica [A] time = 0.191154, size = 159, normalized size = 0.76

$$-\frac{(c\cot(a+bx))^{3/2}\left(8\sqrt{\cot(a+bx)}+\sqrt{2}\log\left(\cot(a+bx)-\sqrt{2}\sqrt{\cot(a+bx)}+1\right)-\sqrt{2}\log\left(\cot(a+bx)+\sqrt{2}\sqrt{\cot(a+b}\right.\right.}{4b\cot^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(3/2),x]
```

```
[Out] -((c*Cot[a + b*x])^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*x]]]
- 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[a + b*x]]] + 8*Sqrt[Cot[a + b*x]] +
 Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[a + b*x]] + Cot[a + b*x]] - Sqrt[2]*Log[1
```

```
 + Sqrt[2]*Sqrt[Cot[a + b*x]] + Cot[a + b*x]]))/(4*b*Cot[a + b*x]^(3/2))
```

Maple [A] time = 0.03, size = 176, normalized size = 0.8

$$-2\,\frac{c\sqrt{c\cot\left(bx+a\right)}}{b}-\frac{c\sqrt{2}}{2\,b}\sqrt[4]{c^2}\arctan\left(-\sqrt{2}\sqrt{c\cot\left(bx+a\right)}\frac{1}{\sqrt[4]{c^2}}+1\right)+\frac{c\sqrt{2}}{4\,b}\sqrt[4]{c^2}\ln\left(\left(c\cot\left(bx+a\right)+\sqrt[4]{c^2}\sqrt{c\cot\left(bx+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cot(b*x+a))^(3/2),x)
```

```
[Out] -2*c*(c*cot(b*x+a))^(1/2)/b-1/2/b*c*(c^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(c^
2)^(1/4)*(c*cot(b*x+a))^(1/2)+1)+1/4/b*c*(c^2)^(1/4)*2^(1/2)*ln((c*cot(b*x+
a)+(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)*2^(1/2)+(c^2)^(1/2))/(c*cot(b*x+a)-(c^2
)^(1/4)*(c*cot(b*x+a))^(1/2)*2^(1/2)+(c^2)^(1/2)))+1/2/b*c*(c^2)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(a+bx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*cot(a + b*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(3/2), x)
```

3.12 $\int \sqrt{c\cot(a+bx)}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{c}\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}+\frac{\sqrt{c}\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}+\frac{\sqrt{c}\tan^{-1}\left(\right.}{}$$

```
[Out] (Sqrt[c]*ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b) -
(Sqrt[c]*ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b) -
(Sqrt[c]*Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]]
)/(2*Sqrt[2]*b) + (Sqrt[c]*Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqr
t[c*Cot[a + b*x]]])/(2*Sqrt[2]*b)
```

Rubi [A] time = 0.11515, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{c}\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}+\frac{\sqrt{c}\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b}+\frac{\sqrt{c}\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Cot[a + b*x]],x]
```

```
[Out] (Sqrt[c]*ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b) -
(Sqrt[c]*ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]])/(Sqrt[2]*b) -
(Sqrt[c]*Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]]
)/(2*Sqrt[2]*b) + (Sqrt[c]*Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqr
t[c*Cot[a + b*x]]])/(2*Sqrt[2]*b)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c\cot(a+bx)}\,dx &= -\frac{c\,\text{Subst}\left(\int \frac{\sqrt{x}}{c^2+x^2}\,dx, x, c\cot(a+bx)\right)}{b}\\
&= -\frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b}\\
&= \frac{c\,\text{Subst}\left(\int \frac{c-x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b} - \frac{c\,\text{Subst}\left(\int \frac{c+x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b}\\
&= -\frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}+2x}{-c-\sqrt{2}\sqrt{c}x-x^2}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}b} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}-2x}{-c+\sqrt{2}\sqrt{c}x-x^2}\,dx, x, \sqrt{c\cot(a+b
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\sqrt{c}\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}b} + \frac{\sqrt{c}\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\,co}\right.}{2\sqrt{2}b}\\
&= \frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}b} - \frac{\sqrt{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}b} - \frac{\sqrt{c}\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)-\right.}{2\sqrt{2}b}
\end{aligned}$$

Mathematica [C] time = 0.0391497, size = 40, normalized size = 0.21

$$-\frac{2(c\cot(a+bx))^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(a+bx)\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Cot[a + b*x]],x]
```

```
[Out] (-2*(c*Cot[a + b*x])^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b*x]^2])
/(3*b*c)
```

Maple [A] time = 0.053, size = 160, normalized size = 0.8

$$-\frac{c\sqrt{2}}{4\,b}\ln\left(\left(c\cot(bx+a)-\sqrt[4]{c^2}\sqrt{c\cot(bx+a)}\sqrt{2}+\sqrt{c^2}\right)\left(c\cot(bx+a)+\sqrt[4]{c^2}\sqrt{c\cot(bx+a)}\sqrt{2}+\sqrt{c^2}\right)^{-1}\right)\frac{1}{\sqrt[4]{c^2}} - \frac{c\sqrt{}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cot(b*x+a))^(1/2),x)
```

```
[Out] -1/4/b*c/(c^2)^(1/4)*2^(1/2)*ln((c*cot(b*x+a)-(c^2)^(1/4)*(c*cot(b*x+a))^(1
/2)*2^(1/2)+(c^2)^(1/2))/(c*cot(b*x+a)+(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)*2^(
1/2)+(c^2)^(1/2)))-1/2/b*c/(c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c^2)^(1/4)*(
c*cot(b*x+a))^(1/2)+1)+1/2/b*c/(c^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(c^2)^(1
/4)*(c*cot(b*x+a))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \cot (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*cot(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \cot (bx + a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*cot(b*x + a)), x)
```

3.13 $\int \frac{1}{\sqrt{c\cot(a+bx)}}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b\sqrt{c}}-\frac{\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b\sqrt{c}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot(a}}{\sqrt{c}}\right.}{\sqrt{2}b\sqrt{c}}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*Sqrt[c]) - Ar
cTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*Sqrt[c]) + Log[
Sqrt[c] + Sqrt[c]*Cot[a + b*x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b
*Sqrt[c]) - Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x
]]]/(2*Sqrt[2]*b*Sqrt[c])
```

Rubi [A] time = 0.112279, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b\sqrt{c}}-\frac{\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}b\sqrt{c}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot(a}}{\sqrt{c}}\right.}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Cot[a + b*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*Sqrt[c]) - Ar
cTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*Sqrt[c]) + Log[
Sqrt[c] + Sqrt[c]*Cot[a + b*x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b
*Sqrt[c]) - Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x
]]]/(2*Sqrt[2]*b*Sqrt[c])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{c\cot(a+bx)}}\,dx = -\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(c^2+x^2\right)}\,dx, x, c\cot(a+bx)\right)}{b}$$

$$= -\frac{(2c)\,\text{Subst}\left(\int \frac{1}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{c-x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{c+x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{c-\sqrt{2}\sqrt{c}x+x^2}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{c+\sqrt{2}\sqrt{c}x+x^2}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{2b} + \frac{\text{Su}}{}$$

$$= \frac{\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}b\sqrt{c}} - \frac{\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}b\sqrt{c}}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a}\right.}{2\sqrt{2}b\sqrt{c}}$$

Mathematica [A] time = 0.0857641, size = 131, normalized size = 0.68

$$\frac{\sqrt{\cot(a+bx)}\left(\log\left(\cot(a+bx)-\sqrt{2}\sqrt{\cot(a+bx)}+1\right)-\log\left(\cot(a+bx)+\sqrt{2}\sqrt{\cot(a+bx)}+1\right)+2\tan^{-1}\left(1-\sqrt{2}\sqrt{\text{co}}\right.\right.}{2\sqrt{2}b\sqrt{c\cot(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Cot[a + b*x]],x]
```

```
[Out] (Sqrt[Cot[a + b*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*x]]] - 2*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[a + b*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[a + b*x]] + Cot[a
+ b*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[a + b*x]] + Cot[a + b*x]]))/(2*Sqrt[2]*b
*Sqrt[c*Cot[a + b*x]])
```

Maple [A] time = 0.064, size = 166, normalized size = 0.9

$$-\frac{\sqrt{2}}{4\,bc}\sqrt[4]{c^2}\ln\left(\left(c\cot(bx+a)+\sqrt[4]{c^2}\sqrt{c\cot(bx+a)}\sqrt{2}+\sqrt{c^2}\right)\left(c\cot(bx+a)-\sqrt[4]{c^2}\sqrt{c\cot(bx+a)}\sqrt{2}+\sqrt{c^2}\right)^{-1}\right)-\frac{\sqrt{2}}{2\,bc}\sqrt[4]{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cot(b*x+a))^(1/2),x)
```

```
[Out] -1/4/b/c*(c^2)^(1/4)*2^(1/2)*ln((c*cot(b*x+a)+(c^2)^(1/4)*(c*cot(b*x+a))^(1
/2)*2^(1/2)+(c^2)^(1/2))/(c*cot(b*x+a)-(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)*2^(
1/2)+(c^2)^(1/2)))-1/2/b/c*(c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c^2)^(1/4)*(
c*cot(b*x+a))^(1/2)+1)+1/2/b/c*(c^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(c^2)^(1
/4)*(c*cot(b*x+a))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c \cot{\left(a + bx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*cot(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\cot(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*cot(b*x + a)), x)
```

3.14 $\int \frac{1}{(c\cot(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{3/2}}-\frac{\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{3/2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot}}{\sqrt{c}}\right.}{\sqrt{2}bc^{3/2}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(3/2))) +
 ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(3/2)) + 2
/(b*c*Sqrt[c*Cot[a + b*x]]) + Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] - Sqrt[2]*
Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b*c^(3/2)) - Log[Sqrt[c] + Sqrt[c]*Cot[a +
 b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b*c^(3/2))
```

Rubi [A] time = 0.140133, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{3/2}}-\frac{\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{3/2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot}}{\sqrt{c}}\right.}{\sqrt{2}bc^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(-3/2),x]
```


```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(3/2))) +
 ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(3/2)) + 2
/(b*c*Sqrt[c*Cot[a + b*x]]) + Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] - Sqrt[2]*
Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b*c^(3/2)) - Log[Sqrt[c] + Sqrt[c]*Cot[a +
 b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b*c^(3/2))
```


Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(c\cot(a+bx))^{3/2}}\,dx = \frac{2}{bc\sqrt{c\cot(a+bx)}} - \frac{\int \sqrt{c\cot(a+bx)}\,dx}{c^2}$$
$$= \frac{2}{bc\sqrt{c\cot(a+bx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{c^2+x^2}\,dx, x, c\cot(a+bx)\right)}{bc}$$
$$= \frac{2}{bc\sqrt{c\cot(a+bx)}} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{bc}$$
$$= \frac{2}{bc\sqrt{c\cot(a+bx)}} - \frac{\text{Subst}\left(\int \frac{c-x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{bc} + \frac{\text{Subst}\left(\int \frac{c+x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+\right.}{bc}$$
$$= \frac{2}{bc\sqrt{c\cot(a+bx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}+2x}{-c-\sqrt{2}\sqrt{c}x-x^2}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}bc^{3/2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}-2x}{-c+\sqrt{2}\sqrt{c}x-x^2}\,dx,\right.}{2\sqrt{2}bc^{3}}$$
$$= \frac{2}{bc\sqrt{c\cot(a+bx)}} + \frac{\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}bc^{3/2}} - \frac{\log\left(\sqrt{c}+\sqrt{c}\cot(a+\right.}{2\sqrt{}}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}bc^{3/2}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}bc^{3/2}} + \frac{2}{bc\sqrt{c\cot(a+bx)}} + \frac{\log\left(\sqrt{c}+\sqrt{c}\right.}{}$$

Mathematica [C] time = 0.0625325, size = 38, normalized size = 0.18

$$\frac{2\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\cot^2(a+bx)\right)}{bc\sqrt{c\cot(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(-3/2),x]
```

```
[Out] (2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[a + b*x]^2])/(b*c*Sqrt[c*Cot[a + b*
x]])
```

Maple [A] time = 0.034, size = 184, normalized size = 0.9

$$2\frac{1}{bc\sqrt{c\cot(bx+a)}}+\frac{\sqrt{2}}{4\,bc}\ln\left(\left(c\cot(bx+a)-\sqrt[4]{c^2}\sqrt{c\cot(bx+a)}\sqrt{2}+\sqrt{c^2}\right)\left(c\cot(bx+a)+\sqrt[4]{c^2}\sqrt{c\cot(bx+a)}\sqrt{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cot(b*x+a))^(3/2),x)
```

```
[Out] 2/b/c/(c*cot(b*x+a))^(1/2)+1/4/b/c/(c^2)^(1/4)*2^(1/2)*ln((c*cot(b*x+a)-(c^
2)^(1/4)*(c*cot(b*x+a))^(1/2)*2^(1/2)+(c^2)^(1/2))/(c*cot(b*x+a)+(c^2)^(1/4
)*(c*cot(b*x+a))^(1/2)*2^(1/2)+(c^2)^(1/2)))+1/2/b/c/(c^2)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)+1)-1/2/b/c/(c^2)^(1/4)*2^(1/2
)*arctan(-2^(1/2)/(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*cot(a + b*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(-3/2), x)
```

3.15 $\int \frac{1}{(c\cot(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{5/2}}+\frac{\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{5/2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot}}{\sqrt{c}}\right.}{\sqrt{2}bc^{5/2}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(5/2))) +
 ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(5/2)) + 2
/(3*b*c*(c*Cot[a + b*x])^(3/2)) - Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] - Sqrt
[2]*Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b*c^(5/2)) + Log[Sqrt[c] + Sqrt[c]*Cot
[a + b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b*c^(5/2))
```

Rubi [A] time = 0.143481, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{5/2}}+\frac{\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{5/2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot}}{\sqrt{c}}\right.}{\sqrt{2}bc^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(-5/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(5/2))) +
 ArcTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(5/2)) + 2
/(3*b*c*(c*Cot[a + b*x])^(3/2)) - Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] - Sqrt
[2]*Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b*c^(5/2)) + Log[Sqrt[c] + Sqrt[c]*Cot
[a + b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b*c^(5/2))
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(c\cot(a+bx))^{5/2}}\,dx &= \frac{2}{3bc(c\cot(a+bx))^{3/2}} - \frac{\int \frac{1}{\sqrt{c\cot(a+bx)}}\,dx}{c^2} \\
&= \frac{2}{3bc(c\cot(a+bx))^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}(c^2+x^2)}\,dx, x, c\cot(a+bx)\right)}{bc} \\
&= \frac{2}{3bc(c\cot(a+bx))^{3/2}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{bc} \\
&= \frac{2}{3bc(c\cot(a+bx))^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{c-x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{bc^2} + \frac{\operatorname{Subst}\left(\int \frac{c+x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a}\right.}{bc^2} \\
&= \frac{2}{3bc(c\cot(a+bx))^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}+2x}{-c-\sqrt{2}\sqrt{c}x-x^2}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}bc^{5/2}} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}-2x}{-c+\sqrt{2}\sqrt{c}x-x^2}\,d\right.}{2\sqrt{2}bc} \\
&= \frac{2}{3bc(c\cot(a+bx))^{3/2}} - \frac{\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}bc^{5/2}} + \frac{\log\left(\sqrt{c}+\sqrt{c}\cot(a\right.}{2} \\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}bc^{5/2}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}bc^{5/2}} + \frac{2}{3bc(c\cot(a+bx))^{3/2}} - \frac{\log\left(\sqrt{c}+\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.069342, size = 40, normalized size = 0.19

$$\frac{2\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\cot^2(a+bx)\right)}{3bc(c\cot(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(-5/2),x]
```

```
[Out] (2*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[a + b*x]^2])/(3*b*c*(c*Cot[a + b*x]
)^(3/2))
```

Maple [A] time = 0.033, size = 184, normalized size = 0.9

$$\frac{2}{3\,bc}\left(c\cot\left(bx+a\right)\right)^{-\frac{3}{2}}+\frac{\sqrt{2}}{4\,bc^{3}}\sqrt[4]{c^{2}}\ln\left(\left(c\cot\left(bx+a\right)+\sqrt[4]{c^{2}}\sqrt{c\cot\left(bx+a\right)}\sqrt{2}+\sqrt{c^{2}}\right)\left(c\cot\left(bx+a\right)-\sqrt[4]{c^{2}}\sqrt{c\cot\left(bx+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cot(b*x+a))^(5/2),x)
```

```
[Out] 2/3/b/c/(c*cot(b*x+a))^(3/2)+1/4/b/c^3*(c^2)^(1/4)*2^(1/2)*ln((c*cot(b*x+a)
+(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)*2^(1/2)+(c^2)^(1/2))/(c*cot(b*x+a)-(c^2)^
(1/4)*(c*cot(b*x+a))^(1/2)*2^(1/2)+(c^2)^(1/2)))+1/2/b/c^3*(c^2)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)+1)-1/2/b/c^3*(c^2)^(1/4
)*2^(1/2)*arctan(-2^(1/2)/(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))**(5/2),x)
```

```
[Out] Integral((c*cot(a + b*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(-5/2), x)
```

3.16 $\int \frac{1}{(c\cot(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{2}{bc^3\sqrt{c\cot(a+bx)}}-\frac{\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{7/2}}+\frac{\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{7/2}}$$

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(7/2)) - Ar
cTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(7/2)) + 2/(5
*b*c*(c*Cot[a + b*x])^(5/2)) - 2/(b*c^3*Sqrt[c*Cot[a + b*x]]) - Log[Sqrt[c]
 + Sqrt[c]*Cot[a + b*x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b*c^(7/2
)) + Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x]]]/(2*
Sqrt[2]*b*c^(7/2))
```

Rubi [A] time = 0.170981, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2}{bc^3\sqrt{c\cot(a+bx)}}-\frac{\log\left(\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{7/2}}+\frac{\log\left(\sqrt{c}\cot(a+bx)+\sqrt{2}\sqrt{c\cot(a+bx)}+\sqrt{c}\right)}{2\sqrt{2}bc^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(-7/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(7/2)) - Ar
cTan[1 + (Sqrt[2]*Sqrt[c*Cot[a + b*x]])/Sqrt[c]]/(Sqrt[2]*b*c^(7/2)) + 2/(5
*b*c*(c*Cot[a + b*x])^(5/2)) - 2/(b*c^3*Sqrt[c*Cot[a + b*x]]) - Log[Sqrt[c]
 + Sqrt[c]*Cot[a + b*x] - Sqrt[2]*Sqrt[c*Cot[a + b*x]]]/(2*Sqrt[2]*b*c^(7/2
)) + Log[Sqrt[c] + Sqrt[c]*Cot[a + b*x] + Sqrt[2]*Sqrt[c*Cot[a + b*x]]]/(2*
Sqrt[2]*b*c^(7/2))
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(c\cot(a+bx))^{7/2}}\,dx = \frac{2}{5bc(c\cot(a+bx))^{5/2}} - \frac{\int \frac{1}{(c\cot(a+bx))^{3/2}}\,dx}{c^2}$$
$$= \frac{2}{5bc(c\cot(a+bx))^{5/2}} - \frac{2}{bc^3\sqrt{c\cot(a+bx)}} + \frac{\int \sqrt{c\cot(a+bx)}\,dx}{c^4}$$
$$= \frac{2}{5bc(c\cot(a+bx))^{5/2}} - \frac{2}{bc^3\sqrt{c\cot(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{x}}{c^2+x^2}\,dx, x, c\cot(a+bx)\right)}{bc^3}$$
$$= \frac{2}{5bc(c\cot(a+bx))^{5/2}} - \frac{2}{bc^3\sqrt{c\cot(a+bx)}} - \frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{bc^3}$$
$$= \frac{2}{5bc(c\cot(a+bx))^{5/2}} - \frac{2}{bc^3\sqrt{c\cot(a+bx)}} + \frac{\mathrm{Subst}\left(\int \frac{c-x^2}{c^2+x^4}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{bc^3} - \frac{\mathrm{Subst}\,(}{}$$
$$= \frac{2}{5bc(c\cot(a+bx))^{5/2}} - \frac{2}{bc^3\sqrt{c\cot(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{c}+2x}{-c-\sqrt{2}\sqrt{c}x-x^2}\,dx, x, \sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}bc^{7/2}} - $$
$$= \frac{2}{5bc(c\cot(a+bx))^{5/2}} - \frac{2}{bc^3\sqrt{c\cot(a+bx)}} - \frac{\log\left(\sqrt{c}+\sqrt{c}\cot(a+bx)-\sqrt{2}\sqrt{c\cot(a+bx)}\right)}{2\sqrt{2}bc^{7/2}}$$
$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}bc^{7/2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c\cot(a+bx)}}{\sqrt{c}}\right)}{\sqrt{2}bc^{7/2}} + \frac{2}{5bc(c\cot(a+bx))^{5/2}} - \frac{2}{bc^3\sqrt{c\cot(a}}$$

Mathematica [C] time = 0.111973, size = 40, normalized size = 0.17

$$\frac{2\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\cot^2(a+bx)\right)}{5bc(c\cot(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(-7/2),x]
```

```
[Out] (2*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[a + b*x]^2])/(5*b*c*(c*Cot[a + b*x
])^(5/2))
```

Maple [A] time = 0.035, size = 202, normalized size = 0.9

$$\frac{2}{5\,bc}\left(c\cot\left(bx+a\right)\right)^{-\frac{5}{2}}-2\,\frac{1}{bc^{3}\sqrt{c\cot\left(bx+a\right)}}-\frac{\sqrt{2}}{4\,bc^{3}}\ln\left(\left(c\cot\left(bx+a\right)-\sqrt[4]{c^{2}}\sqrt{c\cot\left(bx+a\right)}\sqrt{2}+\sqrt{c^{2}}\right)\left(c\cot\left(bx+a\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cot(b*x+a))^(7/2),x)
```

```
[Out] 2/5/b/c/(c*cot(b*x+a))^(5/2)-2/b/c^3/(c*cot(b*x+a))^(1/2)-1/4/b/c^3/(c^2)^(
1/4)*2^(1/2)*ln((c*cot(b*x+a)-(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)*2^(1/2)+(c^2
)^(1/2))/(c*cot(b*x+a)+(c^2)^(1/4)*(c*cot(b*x+a))^(1/2)*2^(1/2)+(c^2)^(1/2)
))-1/2/b/c^3/(c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c^2)^(1/4)*(c*cot(b*x+a))^
(1/2)+1)+1/2/b/c^3/(c^2)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(c^2)^(1/4)*(c*cot(b
*x+a))^(1/2)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(a+bx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))**(7/2),x)
```

```
[Out] Integral((c*cot(a + b*x))**(-7/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(-7/2), x)
```

3.17 $\int (c\cot(a+bx))^{4/3}\,dx$

Optimal. Leaf size=242

$$-\frac{\sqrt{3}c^{4/3}\log\left(-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4b}+\frac{\sqrt{3}c^{4/3}\log\left(\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+\right.}{4b}$$

```
[Out] (c^(4/3)*ArcTan[(c*Cot[a + b*x])^(1/3)/c^(1/3)])/b - (c^(4/3)*ArcTan[Sqrt[3
] - (2*(c*Cot[a + b*x])^(1/3))/c^(1/3)])/(2*b) + (c^(4/3)*ArcTan[Sqrt[3] +
(2*(c*Cot[a + b*x])^(1/3))/c^(1/3)])/(2*b) - (3*c*(c*Cot[a + b*x])^(1/3))/b
 - (Sqrt[3]*c^(4/3)*Log[c^(2/3) - Sqrt[3]*c^(1/3)*(c*Cot[a + b*x])^(1/3) +
(c*Cot[a + b*x])^(2/3)])/(4*b) + (Sqrt[3]*c^(4/3)*Log[c^(2/3) + Sqrt[3]*c^(
1/3)*(c*Cot[a + b*x])^(1/3) + (c*Cot[a + b*x])^(2/3)])/(4*b)
```

Rubi [A] time = 0.469561, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3473, 3476, 329, 209, 634, 618, 204, 628, 203}

$$-\frac{\sqrt{3}c^{4/3}\log\left(-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4b}+\frac{\sqrt{3}c^{4/3}\log\left(\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(4/3),x]
```

```
[Out] (c^(4/3)*ArcTan[(c*Cot[a + b*x])^(1/3)/c^(1/3)])/b - (c^(4/3)*ArcTan[Sqrt[3
] - (2*(c*Cot[a + b*x])^(1/3))/c^(1/3)])/(2*b) + (c^(4/3)*ArcTan[Sqrt[3] +
(2*(c*Cot[a + b*x])^(1/3))/c^(1/3)])/(2*b) - (3*c*(c*Cot[a + b*x])^(1/3))/b
 - (Sqrt[3]*c^(4/3)*Log[c^(2/3) - Sqrt[3]*c^(1/3)*(c*Cot[a + b*x])^(1/3) +
(c*Cot[a + b*x])^(2/3)])/(4*b) + (Sqrt[3]*c^(4/3)*Log[c^(2/3) + Sqrt[3]*c^(
1/3)*(c*Cot[a + b*x])^(1/3) + (c*Cot[a + b*x])^(2/3)])/(4*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (c\cot(a+bx))^{4/3}\,dx = -\frac{3c\sqrt[3]{c\cot(a+bx)}}{b} - c^2\int \frac{1}{(c\cot(a+bx))^{2/3}}\,dx$$

$$= -\frac{3c\sqrt[3]{c\cot(a+bx)}}{b} + \frac{c^3\,\text{Subst}\left(\int \frac{1}{x^{2/3}\left(c^2+x^2\right)}\,dx, x, c\cot(a+bx)\right)}{b}$$

$$= -\frac{3c\sqrt[3]{c\cot(a+bx)}}{b} + \frac{\left(3c^3\right)\text{Subst}\left(\int \frac{1}{c^2+x^6}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{b}$$

$$= -\frac{3c\sqrt[3]{c\cot(a+bx)}}{b} + \frac{c^{4/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{c}-\frac{\sqrt{3}x}{2}}{c^{2/3}-\sqrt{3}\sqrt[3]{c}x+x^2}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{b} + \frac{c^{4/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{c}+\cdots}{c^{2/3}+\sqrt{3}\cdots}\right.}{\cdots}$$

$$= \frac{c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{b} - \frac{3c\sqrt[3]{c\cot(a+bx)}}{b} - \frac{\left(\sqrt{3}c^{4/3}\right)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{c}+2x}{c^{2/3}-\sqrt{3}\sqrt[3]{c}x+x^2}\,dx, x, \sqrt[3]{c\cot(a\cdots}\right.}{4b}$$

$$= \frac{c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{b} - \frac{3c\sqrt[3]{c\cot(a+bx)}}{b} - \frac{\sqrt{3}c^{4/3}\log\left(c^{2/3}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(\cdots\right.}{4b}$$

$$= \frac{c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{b} - \frac{c^{4/3}\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)\right)}{2b} + \frac{c^{4/3}\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{c\cot(a+\cdots}}{\sqrt[3]{c}}\right.\right.}{2b}$$

Mathematica [C] time = 0.0301896, size = 38, normalized size = 0.16

$$\frac{3c\sqrt[3]{c\cot(a+bx)}\left(\text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},-\cot^2(a+bx)\right)-1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(4/3),x]
```

```
[Out] (3*c*(c*Cot[a + b*x])^(1/3)*(-1 + Hypergeometric2F1[1/6, 1, 7/6, -Cot[a + b
*x]^2]))/b
```

Maple [A] time = 0.092, size = 214, normalized size = 0.9

$$-3\,{\frac{c\sqrt[3]{c\cot \left(bx+a \right) }}{b}}-{\frac{c\sqrt{3}}{4\,b}\sqrt[6]{{c}^{2}}\ln \left(\left(c\cot \left(bx+a \right) \right) ^{{\frac{2}{3}}}-\sqrt{3}\sqrt[6]{{c}^{2}}\sqrt[3]{c\cot \left(bx+a \right) }+\sqrt[3]{{c}^{2}} \right) }+{\frac{c}{2\,b}\sqrt[6]{{c}^{2}}\arctan \left(2\,{\frac{\sqrt[3]{c\cot \left(bx+ \right.}}{\sqrt[6]{{c}^{2}}}} \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cot(b*x+a))^(4/3),x)
```

```
[Out] -3*c*(c*cot(b*x+a))^(1/3)/b-1/4/b*c*3^(1/2)*(c^2)^(1/6)*ln((c*cot(b*x+a))^(
2/3)-3^(1/2)*(c^2)^(1/6)*(c*cot(b*x+a))^(1/3)+(c^2)^(1/3))+1/2/b*c*(c^2)^(1
/6)*arctan(2*(c*cot(b*x+a))^(1/3)/(c^2)^(1/6)-3^(1/2))+1/b*c*(c^2)^(1/6)*ar
ctan((c*cot(b*x+a))^(1/3)/(c^2)^(1/6))+1/4/b*c*3^(1/2)*(c^2)^(1/6)*ln((c*co
t(b*x+a))^(2/3)+3^(1/2)*(c^2)^(1/6)*(c*cot(b*x+a))^(1/3)+(c^2)^(1/3))+1/2/b
*c*(c^2)^(1/6)*arctan(2*(c*cot(b*x+a))^(1/3)/(c^2)^(1/6)+3^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(a+bx))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))**(4/3),x)
```

```
[Out] Integral((c*cot(a + b*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(4/3), x)
```

3.18 $\int (c\cot(a+bx))^{2/3}\,dx$

Optimal. Leaf size=225

$$-\frac{\sqrt{3}c^{2/3}\log\left(-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4b}+\frac{\sqrt{3}c^{2/3}\log\left(\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}\right.}{4b}$$

```
[Out] -((c^(2/3)*ArcTan[(c*Cot[a + b*x])^(1/3)/c^(1/3)])/b) + (c^(2/3)*ArcTan[Sqr
t[3] - (2*(c*Cot[a + b*x])^(1/3))/c^(1/3)])/(2*b) - (c^(2/3)*ArcTan[Sqrt[3]
 + (2*(c*Cot[a + b*x])^(1/3))/c^(1/3)])/(2*b) - (Sqrt[3]*c^(2/3)*Log[c^(2/3
) - Sqrt[3]*c^(1/3)*(c*Cot[a + b*x])^(1/3) + (c*Cot[a + b*x])^(2/3)])/(4*b)
 + (Sqrt[3]*c^(2/3)*Log[c^(2/3) + Sqrt[3]*c^(1/3)*(c*Cot[a + b*x])^(1/3) +
(c*Cot[a + b*x])^(2/3)])/(4*b)
```

Rubi [A] time = 0.380465, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 295, 634, 618, 204, 628, 203}

$$-\frac{\sqrt{3}c^{2/3}\log\left(-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4b}+\frac{\sqrt{3}c^{2/3}\log\left(\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(2/3),x]
```

```
[Out] -((c^(2/3)*ArcTan[(c*Cot[a + b*x])^(1/3)/c^(1/3)])/b) + (c^(2/3)*ArcTan[Sqr
t[3] - (2*(c*Cot[a + b*x])^(1/3))/c^(1/3)])/(2*b) - (c^(2/3)*ArcTan[Sqrt[3]
 + (2*(c*Cot[a + b*x])^(1/3))/c^(1/3)])/(2*b) - (Sqrt[3]*c^(2/3)*Log[c^(2/3
) - Sqrt[3]*c^(1/3)*(c*Cot[a + b*x])^(1/3) + (c*Cot[a + b*x])^(2/3)])/(4*b)
 + (Sqrt[3]*c^(2/3)*Log[c^(2/3) + Sqrt[3]*c^(1/3)*(c*Cot[a + b*x])^(1/3) +
(c*Cot[a + b*x])^(2/3)])/(4*b)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (c\cot(a+bx))^{2/3}\,dx = -\frac{c\,\text{Subst}\left(\int \frac{x^{2/3}}{c^2+x^2}\,dx, x, c\cot(a+bx)\right)}{b}$$

$$= -\frac{(3c)\,\text{Subst}\left(\int \frac{x^4}{c^2+x^6}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{b}$$

$$= -\frac{c^{2/3}\,\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{c}}{2}+\frac{\sqrt{3}x}{2}}{c^{2/3}-\sqrt{3}\sqrt[3]{c}x+x^2}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{b} - \frac{c^{2/3}\,\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{c}}{2}-\frac{\sqrt{3}x}{2}}{c^{2/3}+\sqrt{3}\sqrt[3]{c}x+x^2}\,dx, x, \sqrt[3]{c\cot}\right.}{b}$$

$$= -\frac{c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{b} - \frac{\left(\sqrt{3}c^{2/3}\right)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{c}+2x}{c^{2/3}-\sqrt{3}\sqrt[3]{c}x+x^2}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{4b} + \frac{\left(\sqrt{3}c^{2/3}\right.}{}$$

$$= -\frac{c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{b} - \frac{\sqrt{3}c^{2/3}\log\left(c^{2/3}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}\right)}{4b} + \frac{\sqrt{3}}{}$$

$$= -\frac{c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{b} + \frac{c^{2/3}\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)\right)}{2b} - \frac{c^{2/3}\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{c\cot}}{}\right.\right.}{2b}$$

Mathematica [C] time = 0.047033, size = 40, normalized size = 0.18

$$-\frac{3(c\cot(a+bx))^{5/3}\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},-\cot^2(a+bx)\right)}{5bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(2/3),x]
```

```
[Out] (-3*(c*Cot[a + b*x])^(5/3)*Hypergeometric2F1[5/6, 1, 11/6, -Cot[a + b*x]^2]
)/(5*b*c)
```

Maple [A] time = 0.065, size = 203, normalized size = 0.9

$$-\frac{\sqrt{3}}{4\,bc}\left(c^2\right)^{\frac{5}{6}}\ln\left((c\cot(bx+a))^{\frac{2}{3}}-\sqrt{3}\sqrt[6]{c^2}\sqrt[3]{c\cot(bx+a)}+\sqrt[3]{c^2}\right)-\frac{c}{2\,b}\arctan\left(2\frac{\sqrt[3]{c\cot(bx+a)}}{\sqrt[6]{c^2}}-\sqrt{3}\right)\frac{1}{\sqrt[6]{c^2}}-\frac{c}{b}\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cot(b*x+a))^(2/3),x)
```

```
[Out] -1/4/b/c*3^(1/2)*(c^2)^(5/6)*ln((c*cot(b*x+a))^(2/3)-3^(1/2)*(c^2)^(1/6)*(c
*cot(b*x+a))^(1/3)+(c^2)^(1/3))-1/2/b*c/(c^2)^(1/6)*arctan(2*(c*cot(b*x+a))
^(1/3)/(c^2)^(1/6)-3^(1/2))-1/b*c/(c^2)^(1/6)*arctan((c*cot(b*x+a))^(1/3)/(
c^2)^(1/6))+1/4/b/c*3^(1/2)*(c^2)^(5/6)*ln((c*cot(b*x+a))^(2/3)+3^(1/2)*(c^
2)^(1/6)*(c*cot(b*x+a))^(1/3)+(c^2)^(1/3))-1/2/b*c/(c^2)^(1/6)*arctan(2*(c*
cot(b*x+a))^(1/3)/(c^2)^(1/6)+3^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(a+bx))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*cot(a + b*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c\cot(bx+a))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(2/3), x)
```

3.19 $\int \sqrt[3]{c \cot(a+bx)}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt[3]{c}\log\left((c\cot(a+bx))^{2/3}+c^{2/3}\right)}{2b}-\frac{\sqrt[3]{c}\log\left(-c^{2/3}(c\cot(a+bx))^{2/3}+(c\cot(a+bx))^{4/3}+c^{4/3}\right)}{4b}+\frac{\sqrt{3}\sqrt[3]{c}\tan^{-1}\left(\frac{c^{2/3}-2(c\cot(a}{\sqrt{3}c^{2/3}}\right.}{2b}$$

```
[Out] (Sqrt[3]*c^(1/3)*ArcTan[(c^(2/3) - 2*(c*Cot[a + b*x])^(2/3))/(Sqrt[3]*c^(2/
3))])/(2*b) + (c^(1/3)*Log[c^(2/3) + (c*Cot[a + b*x])^(2/3)])/(2*b) - (c^(1
/3)*Log[c^(4/3) - c^(2/3)*(c*Cot[a + b*x])^(2/3) + (c*Cot[a + b*x])^(4/3)])
/(4*b)
```

Rubi [A] time = 0.101623, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3476, 329, 275, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{c}\log\left((c\cot(a+bx))^{2/3}+c^{2/3}\right)}{2b}-\frac{\sqrt[3]{c}\log\left(-c^{2/3}(c\cot(a+bx))^{2/3}+(c\cot(a+bx))^{4/3}+c^{4/3}\right)}{4b}+\frac{\sqrt{3}\sqrt[3]{c}\tan^{-1}\left(\frac{c^{2/3}-2(c\cot(a}{\sqrt{3}c^{2/3}}\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(1/3),x]
```

```
[Out] (Sqrt[3]*c^(1/3)*ArcTan[(c^(2/3) - 2*(c*Cot[a + b*x])^(2/3))/(Sqrt[3]*c^(2/
3))])/(2*b) + (c^(1/3)*Log[c^(2/3) + (c*Cot[a + b*x])^(2/3)])/(2*b) - (c^(1
/3)*Log[c^(4/3) - c^(2/3)*(c*Cot[a + b*x])^(2/3) + (c*Cot[a + b*x])^(4/3)])
/(4*b)
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt[3]{c\cot(a+bx)}\,dx = -\frac{c\,\text{Subst}\left(\int \frac{\sqrt[3]{x}}{c^2+x^2}\,dx, x, c\cot(a+bx)\right)}{b}$$
$$= -\frac{(3c)\,\text{Subst}\left(\int \frac{x^3}{c^2+x^6}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{b}$$
$$= -\frac{(3c)\,\text{Subst}\left(\int \frac{x}{c^2+x^3}\,dx, x, (c\cot(a+bx))^{2/3}\right)}{2b}$$
$$= \frac{\sqrt[3]{c}\,\text{Subst}\left(\int \frac{1}{c^{2/3}+x}\,dx, x, (c\cot(a+bx))^{2/3}\right)}{2b} - \frac{\sqrt[3]{c}\,\text{Subst}\left(\int \frac{c^{2/3}+x}{c^{4/3}-c^{2/3}x+x^2}\,dx, x, (c\cot(a+bx))^{2/3}\right)}{2b}$$
$$= \frac{\sqrt[3]{c}\log\left(c^{2/3}+(c\cot(a+bx))^{2/3}\right)}{2b} - \frac{\sqrt[3]{c}\,\text{Subst}\left(\int \frac{-c^{2/3}+2x}{c^{4/3}-c^{2/3}x+x^2}\,dx, x, (c\cot(a+bx))^{2/3}\right)}{4b} - \frac{(3c)\,\text{Subs}}{}$$
$$= \frac{\sqrt[3]{c}\log\left(c^{2/3}+(c\cot(a+bx))^{2/3}\right)}{2b} - \frac{\sqrt[3]{c}\log\left(c^{4/3}-c^{2/3}(c\cot(a+bx))^{2/3}+(c\cot(a+bx))^{4/3}\right)}{4b} - \frac{(3}{}$$
$$= \frac{\sqrt{3}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2(c\cot(a+bx))^{2/3}}{c^{2/3}}}{\sqrt{3}}\right)}{2b} + \frac{\sqrt[3]{c}\log\left(c^{2/3}+(c\cot(a+bx))^{2/3}\right)}{2b} - \frac{\sqrt[3]{c}\log\left(c^{4/3}-c^{2/3}(c\cot(a+b}{4b}$$

Mathematica [C] time = 0.0395061, size = 40, normalized size = 0.31

$$-\frac{3(c\cot(a+bx))^{4/3}\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\cot^2(a+bx)\right)}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(1/3),x]
```

```
[Out] (-3*(c*Cot[a + b*x])^(4/3)*Hypergeometric2F1[2/3, 1, 5/3, -Cot[a + b*x]^2])
/(4*b*c)
```

Maple [A] time = 0.028, size = 114, normalized size = 0.9

$$\frac{c}{2b}\ln\left((c\cot(bx+a))^{\frac{2}{3}}+\sqrt[3]{c^2}\right)\frac{1}{\sqrt[3]{c^2}} - \frac{c}{4b}\ln\left((c\cot(bx+a))^{\frac{4}{3}}-\sqrt[3]{c^2}(c\cot(bx+a))^{\frac{2}{3}}+\left(c^2\right)^{\frac{2}{3}}\right)\frac{1}{\sqrt[3]{c^2}} - \frac{c\sqrt{3}}{2b}\arctan\left(\frac{\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cot(b*x+a))^(1/3),x)
```

```
[Out] 1/2/b*c/(c^2)^(1/3)*ln((c*cot(b*x+a))^(2/3)+(c^2)^(1/3))-1/4/b*c/(c^2)^(1/3
)*ln((c*cot(b*x+a))^(4/3)-(c^2)^(1/3)*(c*cot(b*x+a))^(2/3)+(c^2)^(2/3))-1/2
/b*c*3^(1/2)/(c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(c^2)^(1/3)*(c*cot(b*x+a))^(
2/3)-1))
```

Maxima [A] time = 1.63484, size = 162, normalized size = 1.24

$$-\frac{\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{c}{\tan(bx+a)}\right)^{\frac{2}{3}}-\left(c^2\right)^{\frac{1}{3}}\right)}{3\left(c^2\right)^{\frac{1}{3}}}\right)}{\left(c^2\right)^{\frac{1}{3}}}+\frac{\log\left(\left(\frac{c}{\tan(bx+a)}\right)^{\frac{4}{3}}-\left(c^2\right)^{\frac{1}{3}}\left(\frac{c}{\tan(bx+a)}\right)^{\frac{2}{3}}+\left(c^2\right)^{\frac{2}{3}}\right)}{\left(c^2\right)^{\frac{1}{3}}}-\frac{2\log\left(\left(\frac{c}{\tan(bx+a)}\right)^{\frac{2}{3}}+\left(c^2\right)^{\frac{1}{3}}\right)}{\left(c^2\right)^{\frac{1}{3}}}\right)c}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(c/tan(b*x + a))^(2/3) - (c^2)^(1/3))
/(c^2)^(1/3))/(c^2)^(1/3) + log((c/tan(b*x + a))^(4/3) - (c^2)^(1/3)*(c/tan
(b*x + a))^(2/3) + (c^2)^(2/3))/(c^2)^(1/3) - 2*log((c/tan(b*x + a))^(2/3)
+ (c^2)^(1/3))/(c^2)^(1/3))*c/b
```

Fricas [B] time = 1.67033, size = 560, normalized size = 4.27

$$-\frac{2\sqrt{3}c^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}c-2\sqrt{3}c^{\frac{1}{3}}\left(\frac{c\cos(2bx+2a)+c}{\sin(2bx+2a)}\right)^{\frac{2}{3}}}{3c}\right)-2c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}+\left(\frac{c\cos(2bx+2a)+c}{\sin(2bx+2a)}\right)^{\frac{2}{3}}\right)+c^{\frac{1}{3}}\log\left(\frac{c^{\frac{4}{3}}\sin(2bx+2a)-c^{\frac{2}{3}}\left(\frac{c\cos(2bx+2a)+c}{\sin(2bx+2a)}\right.}{}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*c^(1/3)*arctan(-1/3*(sqrt(3)*c - 2*sqrt(3)*c^(1/3)*((c*cos(
2*b*x + 2*a) + c)/sin(2*b*x + 2*a))^(2/3))/c) - 2*c^(1/3)*log(c^(2/3) + ((c
*cos(2*b*x + 2*a) + c)/sin(2*b*x + 2*a))^(2/3)) + c^(1/3)*log((c^(4/3)*sin(
2*b*x + 2*a) - c^(2/3)*((c*cos(2*b*x + 2*a) + c)/sin(2*b*x + 2*a))^(2/3)*si
n(2*b*x + 2*a) + (c*cos(2*b*x + 2*a) + c)*((c*cos(2*b*x + 2*a) + c)/sin(2*b
*x + 2*a))^(1/3))/sin(2*b*x + 2*a)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{c \cot (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*cot(a + b*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c \cot (bx + a))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cot(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(1/3), x)
```

3.20 $\int \frac{1}{\sqrt[3]{c\cot(a+bx)}}\,dx$

Optimal. Leaf size=131

$$-\frac{\log\left((c\cot(a+bx))^{2/3}+c^{2/3}\right)}{2b\sqrt[3]{c}}+\frac{\log\left(-c^{2/3}(c\cot(a+bx))^{2/3}+(c\cot(a+bx))^{4/3}+c^{4/3}\right)}{4b\sqrt[3]{c}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{c^{2/3}-2(c\cot(a+bx))^{2/3}}{\sqrt{3}c^{2/3}}\right)}{2b\sqrt[3]{c}}$$

```
[Out] (Sqrt[3]*ArcTan[(c^(2/3) - 2*(c*Cot[a + b*x])^(2/3))/(Sqrt[3]*c^(2/3))])/(2
*b*c^(1/3)) - Log[c^(2/3) + (c*Cot[a + b*x])^(2/3)]/(2*b*c^(1/3)) + Log[c^(
4/3) - c^(2/3)*(c*Cot[a + b*x])^(2/3) + (c*Cot[a + b*x])^(4/3)]/(4*b*c^(1/3
))
```

Rubi [A] time = 0.0963444, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3476, 329, 275, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left((c\cot(a+bx))^{2/3}+c^{2/3}\right)}{2b\sqrt[3]{c}}+\frac{\log\left(-c^{2/3}(c\cot(a+bx))^{2/3}+(c\cot(a+bx))^{4/3}+c^{4/3}\right)}{4b\sqrt[3]{c}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{c^{2/3}-2(c\cot(a+bx))^{2/3}}{\sqrt{3}c^{2/3}}\right)}{2b\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(-1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(c^(2/3) - 2*(c*Cot[a + b*x])^(2/3))/(Sqrt[3]*c^(2/3))])/(2
*b*c^(1/3)) - Log[c^(2/3) + (c*Cot[a + b*x])^(2/3)]/(2*b*c^(1/3)) + Log[c^(
4/3) - c^(2/3)*(c*Cot[a + b*x])^(2/3) + (c*Cot[a + b*x])^(4/3)]/(4*b*c^(1/3
))
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{c\cot(a+bx)}}\,dx = -\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}(c^2+x^2)}\,dx, x, c\cot(a+bx)\right)}{b}$$

$$= -\frac{(3c)\,\text{Subst}\left(\int \frac{x}{c^2+x^6}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{b}$$

$$= -\frac{(3c)\,\text{Subst}\left(\int \frac{1}{c^2+x^3}\,dx, x, (c\cot(a+bx))^{2/3}\right)}{2b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{c^{2/3}+x}\,dx, x, (c\cot(a+bx))^{2/3}\right)}{2b\sqrt[3]{c}} - \frac{\text{Subst}\left(\int \frac{2c^{2/3}-x}{c^{4/3}-c^{2/3}x+x^2}\,dx, x, (c\cot(a+bx))^{2/3}\right)}{2b\sqrt[3]{c}}$$

$$= -\frac{\log\left(c^{2/3}+(c\cot(a+bx))^{2/3}\right)}{2b\sqrt[3]{c}} + \frac{\text{Subst}\left(\int \frac{-c^{2/3}+2x}{c^{4/3}-c^{2/3}x+x^2}\,dx, x, (c\cot(a+bx))^{2/3}\right)}{4b\sqrt[3]{c}} - \frac{\left(3\sqrt[3]{c}\right)\text{Subs}}{}$$

$$= -\frac{\log\left(c^{2/3}+(c\cot(a+bx))^{2/3}\right)}{2b\sqrt[3]{c}} + \frac{\log\left(c^{4/3}-c^{2/3}(c\cot(a+bx))^{2/3}+(c\cot(a+bx))^{4/3}\right)}{4b\sqrt[3]{c}} - \frac{3\,\text{Sub}}{}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2(c\cot(a+bx))^{2/3}}{c^{2/3}}}{\sqrt{3}}\right)}{2b\sqrt[3]{c}} - \frac{\log\left(c^{2/3}+(c\cot(a+bx))^{2/3}\right)}{2b\sqrt[3]{c}} + \frac{\log\left(c^{4/3}-c^{2/3}(c\cot(a+bx))^{2/3}+\right.}{4b\sqrt[3]{c}}$$

Mathematica [A] time = 0.150585, size = 98, normalized size = 0.75

$$\frac{\sqrt[3]{\cot(a+bx)}\left(-2\log\left(\cot^{\frac{2}{3}}(a+bx)+1\right)+\log\left(\cot^{\frac{4}{3}}(a+bx)-\cot^{\frac{2}{3}}(a+bx)+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{2\cot^{\frac{2}{3}}(a+bx)-1}{\sqrt{3}}\right)\right)}{4b\sqrt[3]{c\cot(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(-1/3),x]
```

```
[Out] (Cot[a + b*x]^(1/3)*(-2*Sqrt[3]*ArcTan[(-1 + 2*Cot[a + b*x]^(2/3))/Sqrt[3]]
 - 2*Log[1 + Cot[a + b*x]^(2/3)] + Log[1 - Cot[a + b*x]^(2/3) + Cot[a + b*x
]^(4/3)]))/(4*b*(c*Cot[a + b*x])^(1/3))
```

Maple [A] time = 0.024, size = 114, normalized size = 0.9

$$-\frac{c}{2b}\ln\left(\left(c\cot\left(bx+a\right)\right)^{\frac{2}{3}}+\sqrt[3]{c^{2}}\right)\left(c^{2}\right)^{-\frac{2}{3}}+\frac{c}{4b}\ln\left(\left(c\cot\left(bx+a\right)\right)^{\frac{4}{3}}-\sqrt[3]{c^{2}}\left(c\cot\left(bx+a\right)\right)^{\frac{2}{3}}+\left(c^{2}\right)^{\frac{2}{3}}\right)\left(c^{2}\right)^{-\frac{2}{3}}-\frac{c\sqrt{3}}{2b}\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cot(b*x+a))^(1/3),x)
```

```
[Out] -1/2/b*c/(c^2)^(2/3)*ln((c*cot(b*x+a))^(2/3)+(c^2)^(1/3))+1/4/b*c/(c^2)^(2/
3)*ln((c*cot(b*x+a))^(4/3)-(c^2)^(1/3)*(c*cot(b*x+a))^(2/3)+(c^2)^(2/3))-1/
2/b*c/(c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c^2)^(1/3)*(c*cot(b*x+a))^
(2/3)-1))
```

Maxima [A] time = 1.57281, size = 163, normalized size = 1.24

$$-\frac{c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{c}{\tan\left(bx+a\right)}\right)^{\frac{2}{3}}-\left(c^{2}\right)^{\frac{1}{3}}\right)}{3\left(c^{2}\right)^{\frac{1}{3}}}\right)}{\left(c^{2}\right)^{\frac{2}{3}}}-\frac{\log\left(\left(\frac{c}{\tan\left(bx+a\right)}\right)^{\frac{4}{3}}-\left(c^{2}\right)^{\frac{1}{3}}\left(\frac{c}{\tan\left(bx+a\right)}\right)^{\frac{2}{3}}+\left(c^{2}\right)^{\frac{2}{3}}\right)}{\left(c^{2}\right)^{\frac{2}{3}}}+\frac{2\log\left(\left(\frac{c}{\tan\left(bx+a\right)}\right)^{\frac{2}{3}}+\left(c^{2}\right)^{\frac{1}{3}}\right)}{\left(c^{2}\right)^{\frac{2}{3}}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] -1/4*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(c/tan(b*x + a))^(2/3) - (c^2)^(1/3
))/(c^2)^(1/3))/(c^2)^(2/3) - log((c/tan(b*x + a))^(4/3) - (c^2)^(1/3)*(c/t
an(b*x + a))^(2/3) + (c^2)^(2/3))/(c^2)^(2/3) + 2*log((c/tan(b*x + a))^(2/3
) + (c^2)^(1/3))/(c^2)^(2/3))/b
```

Fricas [B] time = 1.77121, size = 1639, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*c*sqrt((-c)^(1/3)/c)*log(1/2*sqrt(3)*((-c)^(2/3)*((c*cos(2*b*
x + 2*a) + c)/sin(2*b*x + 2*a))^(2/3)*(cos(2*b*x + 2*a) - 1) - 2*c*((c*cos(
2*b*x + 2*a) + c)/sin(2*b*x + 2*a))^(1/3)*sin(2*b*x + 2*a) + (c*cos(2*b*x +
 2*a) - c)*(-c)^(1/3))*sqrt((-c)^(1/3)/c) - 3/2*(-c)^(1/3)*((c*cos(2*b*x +
2*a) + c)/sin(2*b*x + 2*a))^(2/3)*(cos(2*b*x + 2*a) - 1) + 3/2*c*cos(2*b*x
+ 2*a) + 1/2*c) - 2*(-c)^(2/3)*log((-c)^(2/3) + ((c*cos(2*b*x + 2*a) + c)/s
in(2*b*x + 2*a))^(2/3)) + (-c)^(2/3)*log(-((-c)^(1/3)*c*sin(2*b*x + 2*a) +
(-c)^(2/3)*((c*cos(2*b*x + 2*a) + c)/sin(2*b*x + 2*a))^(2/3)*sin(2*b*x + 2*
a) - (c*cos(2*b*x + 2*a) + c)*((c*cos(2*b*x + 2*a) + c)/sin(2*b*x + 2*a))^(
1/3))/sin(2*b*x + 2*a)))/(b*c), -1/4*(2*sqrt(3)*c*sqrt(-(-c)^(1/3)/c)*arcta
n(1/3*(sqrt(3)*(-c)^(1/3)*c*sqrt(-(-c)^(1/3)/c) + 2*sqrt(3)*(-c)^(2/3)*((c*
cos(2*b*x + 2*a) + c)/sin(2*b*x + 2*a))^(2/3)*sqrt(-(-c)^(1/3)/c))/c) + 2*(
-c)^(2/3)*log((-c)^(2/3) + ((c*cos(2*b*x + 2*a) + c)/sin(2*b*x + 2*a))^(2/3
)) - (-c)^(2/3)*log(-((-c)^(1/3)*c*sin(2*b*x + 2*a) + (-c)^(2/3)*((c*cos(2*
b*x + 2*a) + c)/sin(2*b*x + 2*a))^(2/3)*sin(2*b*x + 2*a) - (c*cos(2*b*x + 2
*a) + c)*((c*cos(2*b*x + 2*a) + c)/sin(2*b*x + 2*a))^(1/3))/sin(2*b*x + 2*a
)))/(b*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c \cot{\left(a + bx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*cot(a + b*x))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c \cot\left(bx + a\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(-1/3), x)
```

3.21 $\int \frac{1}{(c\cot(a+bx))^{2/3}}\,dx$

Optimal. Leaf size=225

$$\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4bc^{2/3}}-\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4bc^{2/3}}$$

```
[Out] -(ArcTan[(c*Cot[a + b*x])^(1/3)/c^(1/3)]/(b*c^(2/3))) + ArcTan[Sqrt[3] - (2
*(c*Cot[a + b*x])^(1/3))/c^(1/3)]/(2*b*c^(2/3)) - ArcTan[Sqrt[3] + (2*(c*Co
t[a + b*x])^(1/3))/c^(1/3)]/(2*b*c^(2/3)) + (Sqrt[3]*Log[c^(2/3) - Sqrt[3]*
c^(1/3)*(c*Cot[a + b*x])^(1/3) + (c*Cot[a + b*x])^(2/3)])/(4*b*c^(2/3)) - (
Sqrt[3]*Log[c^(2/3) + Sqrt[3]*c^(1/3)*(c*Cot[a + b*x])^(1/3) + (c*Cot[a + b
*x])^(2/3)])/(4*b*c^(2/3))
```

Rubi [A] time = 0.30955, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3476, 329, 209, 634, 618, 204, 628, 203}

$$\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4bc^{2/3}}-\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4bc^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(-2/3),x]
```

```
[Out] -(ArcTan[(c*Cot[a + b*x])^(1/3)/c^(1/3)]/(b*c^(2/3))) + ArcTan[Sqrt[3] - (2
*(c*Cot[a + b*x])^(1/3))/c^(1/3)]/(2*b*c^(2/3)) - ArcTan[Sqrt[3] + (2*(c*Co
t[a + b*x])^(1/3))/c^(1/3)]/(2*b*c^(2/3)) + (Sqrt[3]*Log[c^(2/3) - Sqrt[3]*
c^(1/3)*(c*Cot[a + b*x])^(1/3) + (c*Cot[a + b*x])^(2/3)])/(4*b*c^(2/3)) - (
Sqrt[3]*Log[c^(2/3) + Sqrt[3]*c^(1/3)*(c*Cot[a + b*x])^(1/3) + (c*Cot[a + b
*x])^(2/3)])/(4*b*c^(2/3))
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(c\cot(a+bx))^{2/3}}\,dx = -\frac{c\,\text{Subst}\left(\int \frac{1}{x^{2/3}(c^2+x^2)}\,dx, x, c\cot(a+bx)\right)}{b}$$

$$= -\frac{(3c)\,\text{Subst}\left(\int \frac{1}{c^2+x^6}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}-\frac{\sqrt{3}x}{2}}{c^{2/3}-\sqrt{3}\sqrt[3]{c}x+x^2}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{bc^{2/3}} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}+\frac{\sqrt{3}x}{2}}{c^{2/3}+\sqrt{3}\sqrt[3]{c}x+x^2}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right.}{bc^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{bc^{2/3}} + \frac{\sqrt{3}\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{c}+2x}{c^{2/3}-\sqrt{3}\sqrt[3]{c}x+x^2}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{4bc^{2/3}} - \frac{\sqrt{3}\,\text{Subst}\left(\int \frac{}{c^{2/3}}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{bc^{2/3}} + \frac{\sqrt{3}\log\left(c^{2/3}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}\right)}{4bc^{2/3}} - \frac{\sqrt{3}\log\left(c\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{bc^{2/3}} + \frac{\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)\right)}{2bc^{2/3}} - \frac{\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)\right)}{2bc^{2/3}} +$$

Mathematica [C] time = 0.0260848, size = 38, normalized size = 0.17

$$-\frac{3\sqrt[3]{c\cot(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},-\cot^2(a+bx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(-2/3),x]

[Out] (-3*(c*Cot[a + b*x])^(1/3)*Hypergeometric2F1[1/6, 1, 7/6, -Cot[a + b*x]^2])
/(b*c)
```

Maple [A] time = 0.059, size = 209, normalized size = 0.9

$$\frac{\sqrt{3}}{4\,bc}\sqrt[6]{c^2}\ln\left((c\cot(bx+a))^{\frac{2}{3}}-\sqrt{3}\sqrt[6]{c^2}\sqrt[3]{c\cot(bx+a)}+\sqrt[3]{c^2}\right)-\frac{1}{2\,bc}\sqrt[6]{c^2}\arctan\left(2\,\frac{\sqrt[3]{c\cot(bx+a)}}{\sqrt[6]{c^2}}-\sqrt{3}\right)-\frac{1}{bc}\sqrt[6]{c^2}\,\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cot(b*x+a))^(2/3),x)
```

```
[Out] 1/4/b/c*3^(1/2)*(c^2)^(1/6)*ln((c*cot(b*x+a))^(2/3)-3^(1/2)*(c^2)^(1/6)*(c*
cot(b*x+a))^(1/3)+(c^2)^(1/3))-1/2/b/c*(c^2)^(1/6)*arctan(2*(c*cot(b*x+a))^
(1/3)/(c^2)^(1/6)-3^(1/2))-1/b/c*(c^2)^(1/6)*arctan((c*cot(b*x+a))^(1/3)/(c
^2)^(1/6))-1/4/b/c*3^(1/2)*(c^2)^(1/6)*ln((c*cot(b*x+a))^(2/3)+3^(1/2)*(c^2
)^(1/6)*(c*cot(b*x+a))^(1/3)+(c^2)^(1/3))-1/2/b/c*(c^2)^(1/6)*arctan(2*(c*c
ot(b*x+a))^(1/3)/(c^2)^(1/6)+3^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c \cot \left(a + b x\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*cot(a + b*x))**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(bx+a))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(-2/3), x)
```

3.22 $\int \frac{1}{(c\cot(a+bx))^{4/3}}\,dx$

Optimal. Leaf size=244

$$\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4bc^{4/3}}-\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4bc^{4/3}}+$$

```
[Out] ArcTan[(c*Cot[a + b*x])^(1/3)/c^(1/3)]/(b*c^(4/3)) - ArcTan[Sqrt[3] - (2*(c
*Cot[a + b*x])^(1/3))/c^(1/3)]/(2*b*c^(4/3)) + ArcTan[Sqrt[3] + (2*(c*Cot[a
 + b*x])^(1/3))/c^(1/3)]/(2*b*c^(4/3)) + 3/(b*c*(c*Cot[a + b*x])^(1/3)) + (
Sqrt[3]*Log[c^(2/3) - Sqrt[3]*c^(1/3)*(c*Cot[a + b*x])^(1/3) + (c*Cot[a + b
*x])^(2/3)])/(4*b*c^(4/3)) - (Sqrt[3]*Log[c^(2/3) + Sqrt[3]*c^(1/3)*(c*Cot[
a + b*x])^(1/3) + (c*Cot[a + b*x])^(2/3)])/(4*b*c^(4/3))
```

Rubi [A] time = 0.424563, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3474, 3476, 329, 295, 634, 618, 204, 628, 203}

$$\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4bc^{4/3}}-\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx))^{2/3}+c^{2/3}\right)}{4bc^{4/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cot[a + b*x])^(-4/3),x]
```

```
[Out] ArcTan[(c*Cot[a + b*x])^(1/3)/c^(1/3)]/(b*c^(4/3)) - ArcTan[Sqrt[3] - (2*(c
*Cot[a + b*x])^(1/3))/c^(1/3)]/(2*b*c^(4/3)) + ArcTan[Sqrt[3] + (2*(c*Cot[a
 + b*x])^(1/3))/c^(1/3)]/(2*b*c^(4/3)) + 3/(b*c*(c*Cot[a + b*x])^(1/3)) + (
Sqrt[3]*Log[c^(2/3) - Sqrt[3]*c^(1/3)*(c*Cot[a + b*x])^(1/3) + (c*Cot[a + b
*x])^(2/3)])/(4*b*c^(4/3)) - (Sqrt[3]*Log[c^(2/3) + Sqrt[3]*c^(1/3)*(c*Cot[
a + b*x])^(1/3) + (c*Cot[a + b*x])^(2/3)])/(4*b*c^(4/3))
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(c\cot(a+bx))^{4/3}}\,dx = \frac{3}{bc\sqrt[3]{c\cot(a+bx)}} - \frac{\int (c\cot(a+bx))^{2/3}\,dx}{c^2}$$

$$= \frac{3}{bc\sqrt[3]{c\cot(a+bx)}} + \frac{\text{Subst}\left(\int \frac{x^{2/3}}{c^2+x^2}\,dx, x, c\cot(a+bx)\right)}{bc}$$

$$= \frac{3}{bc\sqrt[3]{c\cot(a+bx)}} + \frac{3\,\text{Subst}\left(\int \frac{x^4}{c^2+x^6}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{bc}$$

$$= \frac{3}{bc\sqrt[3]{c\cot(a+bx)}} + \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{c}}{2}+\frac{\sqrt{3}x}{2}}{c^{2/3}-\sqrt{3}\sqrt[3]{c}x+x^2}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{bc^{4/3}} + \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{c}}{2}-\frac{\sqrt{3}x}{2}}{c^{2/3}+\sqrt{3}\sqrt[3]{c}x+x^2}\,dx\right.}{bc^{4/3}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{bc^{4/3}} + \frac{3}{bc\sqrt[3]{c\cot(a+bx)}} + \frac{\sqrt{3}\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{c}+2x}{c^{2/3}-\sqrt{3}\sqrt[3]{c}x+x^2}\,dx, x, \sqrt[3]{c\cot(a+bx)}\right)}{4bc^{4/3}} -$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{bc^{4/3}} + \frac{3}{bc\sqrt[3]{c\cot(a+bx)}} + \frac{\sqrt{3}\log\left(c^{2/3}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{c\cot(a+bx)}+(c\cot(a+bx\right.}{4bc^{4/3}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)}{bc^{4/3}} - \frac{\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)\right)}{2bc^{4/3}} + \frac{\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}+\frac{6\sqrt[3]{c\cot(a+bx)}}{\sqrt[3]{c}}\right)\right)}{2bc^{4/3}} + \frac{}{bc}$$

Mathematica [C] time = 0.0569193, size = 38, normalized size = 0.16

$$\frac{3\text{Hypergeometric2F1}\left(-\frac{1}{6},1,\frac{5}{6},-\cot^2(a+bx)\right)}{bc\sqrt[3]{c\cot(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cot[a + b*x])^(-4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1, 5/6, -Cot[a + b*x]^2])/(b*c*(c*Cot[a + b*x])^
(1/3))
```

Maple [A] time = 0.062, size = 229, normalized size = 0.9

$$3\,\frac{1}{bc\sqrt[3]{c\cot(bx+a)}}+\frac{\sqrt{3}}{4\,bc^3}\left(c^2\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\sqrt[6]{c^2}\sqrt[3]{c\cot(bx+a)}-(c\cot(bx+a))^{\frac{2}{3}}-\sqrt[3]{c^2}\right)+\frac{1}{2\,bc}\arctan\left(2\,\frac{\sqrt[3]{c\cot(bx+a}}{\sqrt[6]{c^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cot(b*x+a))^(4/3),x)
```

```
[Out] 3/b/c/(c*cot(b*x+a))^(1/3)+1/4/b/c^3*3^(1/2)*(c^2)^(5/6)*ln(3^(1/2)*(c^2)^(
1/6)*(c*cot(b*x+a))^(1/3)-(c*cot(b*x+a))^(2/3)-(c^2)^(1/3))+1/2/b/c/(c^2)^(
1/6)*arctan(2*(c*cot(b*x+a))^(1/3)/(c^2)^(1/6)-3^(1/2))+1/b/c/(c^2)^(1/6)*a
rctan((c*cot(b*x+a))^(1/3)/(c^2)^(1/6))-1/4/b/c^3*3^(1/2)*(c^2)^(5/6)*ln((c
*cot(b*x+a))^(2/3)+3^(1/2)*(c^2)^(1/6)*(c*cot(b*x+a))^(1/3)+(c^2)^(1/3))+1/
2/b/c/(c^2)^(1/6)*arctan(2*(c*cot(b*x+a))^(1/3)/(c^2)^(1/6)+3^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(a+bx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))**(4/3),x)
```

```
[Out] Integral((c*cot(a + b*x))**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(c\cot(bx+a))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cot(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*cot(b*x + a))^(-4/3), x)
```

3.23 $\int \cot^n(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cot^{n+1}(a+bx)\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\cot^2(a+bx)\right)}{b(n+1)}$$

```
[Out] -((Cot[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Cot[a +
 b*x]^2])/(b*(1 + n)))
```

Rubi [A] time = 0.0279433, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3476, 364}

$$-\frac{\cot^{n+1}(a+bx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^n,x]
```

```
[Out] -((Cot[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Cot[a +
 b*x]^2])/(b*(1 + n)))
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cot^n(a+bx)\,dx = -\frac{\mathrm{Subst}\left(\int \frac{x^n}{1+x^2}\,dx, x, \cot(a+bx)\right)}{b}$$
$$= -\frac{\cot^{1+n}(a+bx)\,{}_2F_1\left(1, \frac{1+n}{2}; \frac{3+n}{2}; -\cot^2(a+bx)\right)}{b(1+n)}$$

Mathematica [A] time = 0.0444441, size = 48, normalized size = 1.04

$$-\frac{\cot^{n+1}(a+bx)\mathrm{Hypergeometric2F1}\left(1, \frac{n+1}{2}, \frac{n+1}{2}+1, -\cot^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^n,x]
```

```
[Out] -((Cot[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, -Cot
[a + b*x]^2])/(b*(1 + n)))
```

Maple [F] time = 0.39, size = 0, normalized size = 0.

$$\int (\cot(bx+a))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^n,x)
```

```
[Out] int(cot(b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cot(bx+a)^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(cot(b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cot\left(bx + a\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cot^{n}\left(a + bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**n,x)
```

```
[Out] Integral(cot(a + b*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cot\left(bx + a\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^n, x)
```

3.24 $\int (b\cot(c+dx))^n\,dx$

Optimal. Leaf size=51

$$-\frac{(b\cot(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\cot^2(c+dx)\right)}{bd(n+1)}$$

```
[Out] -(((b*Cot[c + d*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Cot
[c + d*x]^2])/(b*d*(1 + n)))
```

Rubi [A] time = 0.0318807, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3476, 364}

$$-\frac{(b\cot(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(c+dx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cot[c + d*x])^n,x]
```

```
[Out] -(((b*Cot[c + d*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Cot
[c + d*x]^2])/(b*d*(1 + n)))
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (b\cot(c+dx))^n\,dx = -\frac{b\,\mathrm{Subst}\left(\int \frac{x^n}{b^2+x^2}\,dx, x, b\cot(c+dx)\right)}{d}$$

$$= -\frac{(b\cot(c+dx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\cot^2(c+dx)\right)}{bd(1+n)}$$

Mathematica [A] time = 0.0682271, size = 54, normalized size = 1.06

$$-\frac{\cot(c+dx)(b\cot(c+dx))^n \mathrm{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+1}{2}+1,-\cot^2(c+dx)\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cot[c + d*x])^n,x]
```

```
[Out] -((Cot[c + d*x]*(b*Cot[c + d*x])^n*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 +
 n)/2, -Cot[c + d*x]^2])/(d*(1 + n)))
```

Maple [F] time = 0.378, size = 0, normalized size = 0.

$$\int (b\cot(dx+c))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cot(d*x+c))^n,x)
```

```
[Out] int((b*cot(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cot(dx+c))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cot(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cot(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cot(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cot(c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(d*x+c))**n,x)
```

```
[Out] Integral((b*cot(c + d*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cot(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c))^n, x)
```

3.25 $\int \left(a\cot^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{2}a\cot(x)\sqrt{a\cot^2(x)}-a\tan(x)\sqrt{a\cot^2(x)}\log(\sin(x))$$

```
[Out] -(a*Cot[x]*Sqrt[a*Cot[x]^2])/2 - a*Sqrt[a*Cot[x]^2]*Log[Sin[x]]*Tan[x]
```

Rubi [A] time = 0.0182863, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3658, 3473, 3475}

$$-\frac{1}{2}a\cot(x)\sqrt{a\cot^2(x)}-a\tan(x)\sqrt{a\cot^2(x)}\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x]^2)^(3/2),x]
```

```
[Out] -(a*Cot[x]*Sqrt[a*Cot[x]^2])/2 - a*Sqrt[a*Cot[x]^2]*Log[Sin[x]]*Tan[x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a\cot^2(x)\right)^{3/2}\,dx &= \left(a\sqrt{a\cot^2(x)}\tan(x)\right)\int \cot^3(x)\,dx\\ &= -\frac{1}{2}a\cot(x)\sqrt{a\cot^2(x)} - \left(a\sqrt{a\cot^2(x)}\tan(x)\right)\int \cot(x)\,dx\\ &= -\frac{1}{2}a\cot(x)\sqrt{a\cot^2(x)} - a\sqrt{a\cot^2(x)}\log(\sin(x))\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0193772, size = 27, normalized size = 0.75

$$-\frac{1}{2}a\tan(x)\sqrt{a\cot^2(x)}\left(\csc^2(x)+2\log(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x]^2)^(3/2),x]
```

```
[Out] -(a*Sqrt[a*Cot[x]^2]*(Csc[x]^2 + 2*Log[Sin[x]])*Tan[x])/2
```

Maple [A] time = 0.055, size = 29, normalized size = 0.8

$$\frac{-\left(\cot\left(x\right)\right)^{2}+\ln\left(\left(\cot\left(x\right)\right)^{2}+1\right)}{2\,\left(\cot\left(x\right)\right)^{3}}\left(a\left(\cot\left(x\right)\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)^2)^(3/2),x)
```

```
[Out] 1/2*(a*cot(x)^2)^(3/2)*(-cot(x)^2+ln(cot(x)^2+1))/cot(x)^3
```

Maxima [A] time = 1.54505, size = 41, normalized size = 1.14

$$\frac{1}{2}\,a^{\frac{3}{2}}\log\left(\tan\left(x\right)^{2}+1\right)-a^{\frac{3}{2}}\log\left(\tan\left(x\right)\right)-\frac{a^{\frac{3}{2}}}{2\,\tan\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^(3/2)*log(tan(x)^2 + 1) - a^(3/2)*log(tan(x)) - 1/2*a^(3/2)/tan(x)^2
```

Fricas [A] time = 1.62114, size = 139, normalized size = 3.86

$$\frac{\left((a\cos(2x) - a)\log\left(-\frac{1}{2}\cos(2x) + \frac{1}{2}\right) - 2a\right)\sqrt{-\frac{a\cos(2x)+a}{\cos(2x)-1}}}{2\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a*cos(2*x) - a)*log(-1/2*cos(2*x) + 1/2) - 2*a)*sqrt(-(a*cos(2*x) + a
)/(cos(2*x) - 1))/sin(2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot^2(x)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)**2)**(3/2),x)
```

```
[Out] Integral((a*cot(x)**2)**(3/2), x)
```

Giac [A] time = 1.20721, size = 42, normalized size = 1.17

$$\frac{1}{2}a^{\frac{3}{2}}\left(\frac{1}{\cos(x)^2 - 1} - \log\left(-\cos(x)^2 + 1\right)\right)\mathrm{sgn}(\cos(x))\,\mathrm{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*a^(3/2)*(1/(cos(x)^2 - 1) - log(-cos(x)^2 + 1))*sgn(cos(x))*sgn(sin(x))
```

3.26 $\int \sqrt{a \cot^2(x)}\, dx$

Optimal. Leaf size=16

$$\tan(x)\sqrt{a \cot^2(x)} \log(\sin(x))$$

```
[Out] Sqrt[a*Cot[x]^2]*Log[Sin[x]]*Tan[x]
```

Rubi [A] time = 0.0205653, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3658, 3475}

$$\tan(x)\sqrt{a \cot^2(x)} \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cot[x]^2],x]
```

```
[Out] Sqrt[a*Cot[x]^2]*Log[Sin[x]]*Tan[x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{a \cot^2(x)}\, dx &= \left(\sqrt{a \cot^2(x)} \tan(x)\right) \int \cot(x)\, dx \\ &= \sqrt{a \cot^2(x)} \log(\sin(x)) \tan(x) \end{aligned}$$

Mathematica [A] time = 0.0066614, size = 16, normalized size = 1.

$$\tan(x)\sqrt{a\cot^2(x)}\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cot[x]^2],x]
```

```
[Out] Sqrt[a*Cot[x]^2]*Log[Sin[x]]*Tan[x]
```

Maple [A] time = 0.073, size = 22, normalized size = 1.4

$$-\frac{\ln\left((\cot(x))^2+1\right)}{2\cot(x)}\sqrt{a(\cot(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)^2)^(1/2),x)
```

```
[Out] -1/2*(a*cot(x)^2)^(1/2)/cot(x)*ln(cot(x)^2+1)
```

Maxima [A] time = 1.62628, size = 27, normalized size = 1.69

$$-\frac{1}{2}\sqrt{a}\log\left(\tan(x)^2+1\right)+\sqrt{a}\log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*log(tan(x)^2 + 1) + sqrt(a)*log(tan(x))
```

Fricas [B] time = 1.64949, size = 126, normalized size = 7.88

$$\frac{\sqrt{-\frac{a\cos(2x)+a}{\cos(2x)-1}}\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)\sin(2x)}{2(\cos(2x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-(a*cos(2*x) + a)/(cos(2*x) - 1))*log(-1/2*cos(2*x) + 1/2)*sin(2*x
)/(cos(2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \cot^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*cot(x)**2), x)
```

Giac [A] time = 1.24426, size = 27, normalized size = 1.69

$$\frac{1}{2} \sqrt{a} \log\left(-\cos(x)^2 + 1\right) \operatorname{sgn}(\cos(x)) \operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*log(-cos(x)^2 + 1)*sgn(cos(x))*sgn(sin(x))
```

3.27 $\int \frac{1}{\sqrt{a \cot^2(x)}} dx$

Optimal. Leaf size=17

$$-\frac{\cot(x)\log(\cos(x))}{\sqrt{a\cot^2(x)}}$$

```
[Out] -((Cot[x]*Log[Cos[x]])/Sqrt[a*Cot[x]^2])
```

Rubi [A] time = 0.0117983, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3658, 3475}

$$-\frac{\cot(x)\log(\cos(x))}{\sqrt{a\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cot[x]^2],x]
```

```
[Out] -((Cot[x]*Log[Cos[x]])/Sqrt[a*Cot[x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cot^2(x)}}\,dx = \frac{\cot(x)\int \tan(x)\,dx}{\sqrt{a\cot^2(x)}} = -\frac{\cot(x)\log(\cos(x))}{\sqrt{a\cot^2(x)}}$$

Mathematica [A] time = 0.007814, size = 17, normalized size = 1.

$$-\frac{\cot(x)\log(\cos(x))}{\sqrt{a\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cot[x]^2],x]
```

```
[Out] -((Cot[x]*Log[Cos[x]])/Sqrt[a*Cot[x]^2])
```

Maple [A] time = 0.086, size = 28, normalized size = 1.7

$$-\frac{\cot(x)\left(-\ln\left((\cot(x))^2+1\right)+2\ln(\cot(x))\right)}{2}\frac{1}{\sqrt{a(\cot(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)^2)^(1/2),x)
```

```
[Out] -1/2*cot(x)*(-ln(cot(x)^2+1)+2*ln(cot(x)))/(a*cot(x)^2)^(1/2)
```

Maxima [A] time = 1.55324, size = 16, normalized size = 0.94

$$\frac{\log\left(\tan(x)^2+1\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x)^2 + 1)/sqrt(a)
```

Fricas [B] time = 1.67252, size = 128, normalized size = 7.53

$$-\frac{\sqrt{-\frac{a\cos(2x)+a}{\cos(2x)-1}}\log\left(\frac{1}{2}\cos\left(2x\right)+\frac{1}{2}\right)\sin\left(2x\right)}{2\left(a\cos\left(2x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-(a*cos(2*x) + a)/(cos(2*x) - 1))*log(1/2*cos(2*x) + 1/2)*sin(2*x
)/(a*cos(2*x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\cot^{2}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cot(x)**2), x)
```

Giac [A] time = 1.26688, size = 26, normalized size = 1.53

$$-\frac{\log\left(\left|\cos\left(x\right)\right|\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(x\right)\right)\operatorname{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(cos(x)))/(sqrt(a)*sgn(cos(x))*sgn(sin(x)))
```

3.28 $\int \frac{1}{\left(a\cot^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{\tan(x)}{2a\sqrt{a\cot^2(x)}}+\frac{\cot(x)\log(\cos(x))}{a\sqrt{a\cot^2(x)}}$$

```
[Out] (Cot[x]*Log[Cos[x]])/(a*Sqrt[a*Cot[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cot[x]^2])
```

Rubi [A] time = 0.0181165, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3658, 3473, 3475}

$$\frac{\tan(x)}{2a\sqrt{a\cot^2(x)}}+\frac{\cot(x)\log(\cos(x))}{a\sqrt{a\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x]^2)^(-3/2),x]
```

```
[Out] (Cot[x]*Log[Cos[x]])/(a*Sqrt[a*Cot[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cot[x]^2])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a\cot^2(x)\right)^{3/2}}\,dx &= \frac{\cot(x)\int \tan^3(x)\,dx}{a\sqrt{a\cot^2(x)}}\\
&= \frac{\tan(x)}{2a\sqrt{a\cot^2(x)}} - \frac{\cot(x)\int \tan(x)\,dx}{a\sqrt{a\cot^2(x)}}\\
&= \frac{\cot(x)\log(\cos(x))}{a\sqrt{a\cot^2(x)}} + \frac{\tan(x)}{2a\sqrt{a\cot^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.027751, size = 30, normalized size = 0.77

$$\frac{\csc(x)\sec(x)+2\cot(x)\log(\cos(x))}{2a\sqrt{a\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x]^2)^(-3/2),x]
```

```
[Out] (2*Cot[x]*Log[Cos[x]] + Csc[x]*Sec[x])/(2*a*Sqrt[a*Cot[x]^2])
```

Maple [A] time = 0.046, size = 36, normalized size = 0.9

$$-\frac{\cot(x)\left(\ln\left((\cot(x))^2+1\right)(\cot(x))^2-2\,\ln(\cot(x))(\cot(x))^2-1\right)}{2}\left(a(\cot(x))^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)^2)^(3/2),x)
```

```
[Out] -1/2*cot(x)*(ln(cot(x)^2+1)*cot(x)^2-2*ln(cot(x))*cot(x)^2-1)/(a*cot(x)^2)^
(3/2)
```

Maxima [A] time = 1.5459, size = 30, normalized size = 0.77

$$\frac{\tan(x)^2}{2a^{\frac{3}{2}}} - \frac{\log\left(\tan(x)^2 + 1\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)^2/a^(3/2) - 1/2*log(tan(x)^2 + 1)/a^(3/2)
```

Fricas [B] time = 1.68903, size = 198, normalized size = 5.08

$$\frac{\left((\cos(2x) + 1)\log\left(\frac{1}{2}\cos(2x) + \frac{1}{2}\right)\sin(2x) + 2\sin(2x)\right)\sqrt{-\frac{a\cos(2x)+a}{\cos(2x)-1}}}{2\left(a^2\cos(2x)^2 + 2a^2\cos(2x) + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((cos(2*x) + 1)*log(1/2*cos(2*x) + 1/2)*sin(2*x) + 2*sin(2*x))*sqrt(-(a
*cos(2*x) + a)/(cos(2*x) - 1))/(a^2*cos(2*x)^2 + 2*a^2*cos(2*x) + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cot^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)**2)**(3/2),x)
```

```
[Out] Integral((a*cot(x)**2)**(-3/2), x)
```

Giac [B] time = 1.53959, size = 188, normalized size = 4.82

$$-\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,x\right)^{2}+\frac{1}{\tan\left(\frac{1}{2}\,x\right)^{2}}+2\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)\right)}{\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)}-\frac{\log\left(\tan\left(\frac{1}{2}\,x\right)^{2}+\frac{1}{\tan\left(\frac{1}{2}\,x\right)^{2}}-2\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)\right)}{\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)}+\frac{\left(\tan\left(\frac{1}{2}\,x\right)^{2}+\frac{1}{\tan\left(\frac{1}{2}\,x\right)^{2}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)\right)-6\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right.\right.}{\left(\tan\left(\frac{1}{2}\,x\right)^{2}+\frac{1}{\tan\left(\frac{1}{2}\,x\right)^{2}}-2\right)\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)}}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(log(tan(1/2*x)^2 + 1/tan(1/2*x)^2 + 2)*sgn(tan(1/2*x))/sgn(-tan(1/2*x
)^2 + 1) - log(tan(1/2*x)^2 + 1/tan(1/2*x)^2 - 2)*sgn(tan(1/2*x))/sgn(-tan(
1/2*x)^2 + 1) + ((tan(1/2*x)^2 + 1/tan(1/2*x)^2)*sgn(tan(1/2*x)) - 6*sgn(ta
n(1/2*x)))/((tan(1/2*x)^2 + 1/tan(1/2*x)^2 - 2)*sgn(-tan(1/2*x)^2 + 1)) - s
gn(tan(1/2*x)))/a^(3/2)
```

3.29 $\int \left(a\cot^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=200

$$-\frac{2}{7}a\cot^2(x)\sqrt{a\cot^3(x)}+\frac{2}{3}a\sqrt{a\cot^3(x)}-\frac{a\sqrt{a\cot^3(x)}\log\left(\cot(x)-\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}\cot^{\frac{3}{2}}(x)}+\frac{a\sqrt{a\cot^3(x)}\log\left(\cot(x)+\sqrt{2}\sqrt{\cdots}\right.}{2\sqrt{2}\cot^{\frac{3}{2}}(x)}$$

```
[Out] (2*a*Sqrt[a*Cot[x]^3])/3 + (a*ArcTan[1 - Sqrt[2]*Sqrt[Cot[x]]]*Sqrt[a*Cot[x
]^3])/(Sqrt[2]*Cot[x]^(3/2)) - (a*ArcTan[1 + Sqrt[2]*Sqrt[Cot[x]]]*Sqrt[a*C
ot[x]^3])/(Sqrt[2]*Cot[x]^(3/2)) - (2*a*Cot[x]^2*Sqrt[a*Cot[x]^3])/7 - (a*S
qrt[a*Cot[x]^3]*Log[1 - Sqrt[2]*Sqrt[Cot[x]] + Cot[x]])/(2*Sqrt[2]*Cot[x]^(
3/2)) + (a*Sqrt[a*Cot[x]^3]*Log[1 + Sqrt[2]*Sqrt[Cot[x]] + Cot[x]])/(2*Sqrt
[2]*Cot[x]^(3/2))
```

Rubi [A] time = 0.097225, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {3658, 3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2}{7}a\cot^2(x)\sqrt{a\cot^3(x)}+\frac{2}{3}a\sqrt{a\cot^3(x)}-\frac{a\sqrt{a\cot^3(x)}\log\left(\cot(x)-\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}\cot^{\frac{3}{2}}(x)}+\frac{a\sqrt{a\cot^3(x)}\log\left(\cot(x)+\sqrt{2}\sqrt{\cdots}\right.}{2\sqrt{2}\cot^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x]^3)^(3/2),x]
```


```
[Out] (2*a*Sqrt[a*Cot[x]^3])/3 + (a*ArcTan[1 - Sqrt[2]*Sqrt[Cot[x]]]*Sqrt[a*Cot[x
]^3])/(Sqrt[2]*Cot[x]^(3/2)) - (a*ArcTan[1 + Sqrt[2]*Sqrt[Cot[x]]]*Sqrt[a*C
ot[x]^3])/(Sqrt[2]*Cot[x]^(3/2)) - (2*a*Cot[x]^2*Sqrt[a*Cot[x]^3])/7 - (a*S
qrt[a*Cot[x]^3]*Log[1 - Sqrt[2]*Sqrt[Cot[x]] + Cot[x]])/(2*Sqrt[2]*Cot[x]^(
3/2)) + (a*Sqrt[a*Cot[x]^3]*Log[1 + Sqrt[2]*Sqrt[Cot[x]] + Cot[x]])/(2*Sqrt
[2]*Cot[x]^(3/2))
```


Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \left(a\cot^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\cot^3(x)}\right)\int \cot^{\frac{9}{2}}(x)\,dx}{\cot^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{7}a\cot^2(x)\sqrt{a\cot^3(x)} - \frac{\left(a\sqrt{a\cot^3(x)}\right)\int \cot^{\frac{5}{2}}(x)\,dx}{\cot^{\frac{3}{2}}(x)}$$

$$= \frac{2}{3}a\sqrt{a\cot^3(x)} - \frac{2}{7}a\cot^2(x)\sqrt{a\cot^3(x)} + \frac{\left(a\sqrt{a\cot^3(x)}\right)\int \sqrt{\cot(x)}\,dx}{\cot^{\frac{3}{2}}(x)}$$

$$= \frac{2}{3}a\sqrt{a\cot^3(x)} - \frac{2}{7}a\cot^2(x)\sqrt{a\cot^3(x)} - \frac{\left(a\sqrt{a\cot^3(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \cot(x)\right)}{\cot^{\frac{3}{2}}(x)}$$

$$= \frac{2}{3}a\sqrt{a\cot^3(x)} - \frac{2}{7}a\cot^2(x)\sqrt{a\cot^3(x)} - \frac{\left(2a\sqrt{a\cot^3(x)}\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\cot(x)}\right)}{\cot^{\frac{3}{2}}(x)}$$

$$= \frac{2}{3}a\sqrt{a\cot^3(x)} - \frac{2}{7}a\cot^2(x)\sqrt{a\cot^3(x)} + \frac{\left(a\sqrt{a\cot^3(x)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(x)}\right)}{\cot^{\frac{3}{2}}(x)} - \frac{\left(a\sqrt{a\,\mathrm{c}\right.}{}$$

$$= \frac{2}{3}a\sqrt{a\cot^3(x)} - \frac{2}{7}a\cot^2(x)\sqrt{a\cot^3(x)} - \frac{\left(a\sqrt{a\cot^3(x)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\cot(x)}\right)}{2\cot^{\frac{3}{2}}(x)} - \frac{\left(a\right.}{}$$

$$= \frac{2}{3}a\sqrt{a\cot^3(x)} - \frac{2}{7}a\cot^2(x)\sqrt{a\cot^3(x)} - \frac{a\sqrt{a\cot^3(x)}\log\left(1-\sqrt{2}\sqrt{\cot(x)}+\cot(x)\right)}{2\sqrt{2}\cot^{\frac{3}{2}}(x)} + \frac{a\sqrt{a\cot^3}}{}$$

$$= \frac{2}{3}a\sqrt{a\cot^3(x)} + \frac{a\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(x)}\right)\sqrt{a\cot^3(x)}}{\sqrt{2}\cot^{\frac{3}{2}}(x)} - \frac{a\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(x)}\right)\sqrt{a\cot^3(x)}}{\sqrt{2}\cot^{\frac{3}{2}}(x)} -$$

Mathematica [C] time = 0.0547907, size = 39, normalized size = 0.2

$$-\frac{2}{21}a\sqrt{a\cot^3(x)}\left(7\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(x)\right)+3\cot^2(x)-7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x]^3)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a*Cot[x]^3]*(-7 + 3*Cot[x]^2 + 7*Hypergeometric2F1[3/4, 1, 7/4,
-Cot[x]^2]))/21
```

Maple [A] time = 0.062, size = 189, normalized size = 0.9

$$-\frac{1}{84\left(\cot(x)\right)^{3}a^{2}}\left(a\left(\cot(x)\right)^{3}\right)^{\frac{3}{2}}\left(24\left(a\cot(x)\right)^{7/2}\sqrt[4]{a^{2}}+21\,a^{4}\sqrt{2}\ln\left(-\frac{\sqrt[4]{a^{2}}\sqrt{a\cot(x)}\sqrt{2}-a\cot(x)-\sqrt{a^{2}}}{a\cot(x)+\sqrt[4]{a^{2}}\sqrt{a\cot(x)}\sqrt{2}+\sqrt{a^{2}}}\right)+42\,a^{4}\sqrt{2}\,\mathrm{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)^3)^(3/2),x)
```

```
[Out] -1/84*(a*cot(x)^3)^(3/2)*(24*(a*cot(x))^(7/2)*(a^2)^(1/4)+21*a^4*2^(1/2)*ln
(-((a^2)^(1/4)*(a*cot(x))^(1/2)*2^(1/2)-a*cot(x)-(a^2)^(1/2))/(a*cot(x)+(a^
2)^(1/4)*(a*cot(x))^(1/2)*2^(1/2)+(a^2)^(1/2)))+42*a^4*2^(1/2)*arctan((2^(1
/2)*(a*cot(x))^(1/2)+(a^2)^(1/4))/(a^2)^(1/4))+42*a^4*2^(1/2)*arctan((2^(1/
2)*(a*cot(x))^(1/2)-(a^2)^(1/4))/(a^2)^(1/4))-56*(a*cot(x))^(3/2)*a^2*(a^2)
^(1/4))/cot(x)^3/(a*cot(x))^(3/2)/a^2/(a^2)^(1/4)
```

Maxima [A] time = 1.56199, size = 153, normalized size = 0.76

$$\frac{1}{4}\left(2\sqrt{2}\sqrt{a}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+2\sqrt{2}\sqrt{a}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)+\sqrt{2}\sqrt{a}\log\left(\sqrt{2}\sqrt{\tan(x)}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 2*s
qrt(2)*sqrt(a)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) + sqrt(2)*sq
rt(a)*log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - sqrt(2)*sqrt(a)*log(-sqrt(2)
*sqrt(tan(x)) + tan(x) + 1))*a + 2/3*a^(3/2)/tan(x)^(3/2) - 2/7*a^(3/2)/tan
(x)^(7/2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot^3(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)**3)**(3/2),x)
```

```
[Out] Integral((a*cot(x)**3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(x)^3)^(3/2), x)
```

3.30 $\int \sqrt{a \cot^3(x)}\, dx$

Optimal. Leaf size=176

$$-\frac{\sqrt{a\cot^3(x)}\log\left(\cot(x)-\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}\cot^{\frac{3}{2}}(x)}+\frac{\sqrt{a\cot^3(x)}\log\left(\cot(x)+\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}\cot^{\frac{3}{2}}(x)}-\frac{\sqrt{a\cot^3(x)}\tan^{-1}\left(1-\sqrt{2}\sqrt{co\right.}}{\sqrt{2}\cot^{\frac{3}{2}}(x)}$$

```
[Out] -((ArcTan[1 - Sqrt[2]*Sqrt[Cot[x]]]*Sqrt[a*Cot[x]^3])/(Sqrt[2]*Cot[x]^(3/2)
)) + (ArcTan[1 + Sqrt[2]*Sqrt[Cot[x]]]*Sqrt[a*Cot[x]^3])/(Sqrt[2]*Cot[x]^(3
/2)) - (Sqrt[a*Cot[x]^3]*Log[1 - Sqrt[2]*Sqrt[Cot[x]] + Cot[x]])/(2*Sqrt[2]
*Cot[x]^(3/2)) + (Sqrt[a*Cot[x]^3]*Log[1 + Sqrt[2]*Sqrt[Cot[x]] + Cot[x]])/
(2*Sqrt[2]*Cot[x]^(3/2)) - 2*Sqrt[a*Cot[x]^3]*Tan[x]
```

Rubi [A] time = 0.0872821, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {3658, 3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt{a\cot^3(x)}\log\left(\cot(x)-\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}\cot^{\frac{3}{2}}(x)}+\frac{\sqrt{a\cot^3(x)}\log\left(\cot(x)+\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}\cot^{\frac{3}{2}}(x)}-\frac{\sqrt{a\cot^3(x)}\tan^{-1}\left(1-\sqrt{2}\sqrt{co\right.}}{\sqrt{2}\cot^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cot[x]^3],x]
```

```
[Out] -((ArcTan[1 - Sqrt[2]*Sqrt[Cot[x]]]*Sqrt[a*Cot[x]^3])/(Sqrt[2]*Cot[x]^(3/2)
)) + (ArcTan[1 + Sqrt[2]*Sqrt[Cot[x]]]*Sqrt[a*Cot[x]^3])/(Sqrt[2]*Cot[x]^(3
/2)) - (Sqrt[a*Cot[x]^3]*Log[1 - Sqrt[2]*Sqrt[Cot[x]] + Cot[x]])/(2*Sqrt[2]
*Cot[x]^(3/2)) + (Sqrt[a*Cot[x]^3]*Log[1 + Sqrt[2]*Sqrt[Cot[x]] + Cot[x]])/
(2*Sqrt[2]*Cot[x]^(3/2)) - 2*Sqrt[a*Cot[x]^3]*Tan[x]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a\cot^3(x)}\,dx = \frac{\sqrt{a\cot^3(x)}\int \cot^{\frac{3}{2}}(x)\,dx}{\cot^{\frac{3}{2}}(x)}$$

$$= -2\sqrt{a\cot^3(x)}\tan(x) - \frac{\sqrt{a\cot^3(x)}\int \frac{1}{\sqrt{\cot(x)}}\,dx}{\cot^{\frac{3}{2}}(x)}$$

$$= -2\sqrt{a\cot^3(x)}\tan(x) + \frac{\sqrt{a\cot^3(x)}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \cot(x)\right)}{\cot^{\frac{3}{2}}(x)}$$

$$= -2\sqrt{a\cot^3(x)}\tan(x) + \frac{\left(2\sqrt{a\cot^3(x)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\cot(x)}\right)}{\cot^{\frac{3}{2}}(x)}$$

$$= -2\sqrt{a\cot^3(x)}\tan(x) + \frac{\sqrt{a\cot^3(x)}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(x)}\right)}{\cot^{\frac{3}{2}}(x)} + \frac{\sqrt{a\cot^3(x)}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,\right.}{\cot^{\frac{3}{2}}(x)}$$

$$= -2\sqrt{a\cot^3(x)}\tan(x) + \frac{\sqrt{a\cot^3(x)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\cot(x)}\right)}{2\cot^{\frac{3}{2}}(x)} + \frac{\sqrt{a\cot^3(x)}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\cdots}\right.}{2\cot^{\frac{3}{2}}(\cdots}$$

$$= -\frac{\sqrt{a\cot^3(x)}\log\left(1-\sqrt{2}\sqrt{\cot(x)}+\cot(x)\right)}{2\sqrt{2}\cot^{\frac{3}{2}}(x)} + \frac{\sqrt{a\cot^3(x)}\log\left(1+\sqrt{2}\sqrt{\cot(x)}+\cot(x)\right)}{2\sqrt{2}\cot^{\frac{3}{2}}(x)} - 2\sqrt{a\,\text{co}\cdots}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(x)}\right)\sqrt{a\cot^3(x)}}{\sqrt{2}\cot^{\frac{3}{2}}(x)} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(x)}\right)\sqrt{a\cot^3(x)}}{\sqrt{2}\cot^{\frac{3}{2}}(x)} - \frac{\sqrt{a\cot^3(x)}\log\left(1-\cdots\right.}{2\sqrt{2}\,\text{c}\cdots}$$

<u>Mathematica</u> [A] time = 0.104914, size = 122, normalized size = 0.69

$$-\frac{\sqrt{a\cot^3(x)}\left(8\sqrt{\cot(x)}+\sqrt{2}\log\left(\cot(x)-\sqrt{2}\sqrt{\cot(x)}+1\right)-\sqrt{2}\log\left(\cot(x)+\sqrt{2}\sqrt{\cot(x)}+1\right)+2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\cdots\right.\right.}{4\cot^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cot[x]^3],x]
```

```
[Out] -(Sqrt[a*Cot[x]^3]*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[x]]] - 2*Sqrt[2]*
ArcTan[1 + Sqrt[2]*Sqrt[Cot[x]]] + 8*Sqrt[Cot[x]] + Sqrt[2]*Log[1 - Sqrt[2]
*Sqrt[Cot[x]] + Cot[x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[x]] + Cot[x]]))/
```

```
(4*Cot[x]^(3/2))
```

Maple [A] time = 0.079, size = 165, normalized size = 0.9

$$\frac{1}{4\cot(x)}\sqrt{a(\cot(x))^3}\left(\sqrt[4]{a^2}\sqrt{2}\ln\left(-\left(a\cot(x)+\sqrt[4]{a^2}\sqrt{a\cot(x)}\sqrt{2}+\sqrt{a^2}\right)\left(\sqrt[4]{a^2}\sqrt{a\cot(x)}\sqrt{2}-a\cot(x)-\sqrt{a^2}\right)^{-1}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)^3)^(1/2),x)
```

```
[Out] 1/4*(a*cot(x)^3)^(1/2)*((a^2)^(1/4)*2^(1/2)*ln(-(a*cot(x)+(a^2)^(1/4)*(a*co
t(x))^(1/2)*2^(1/2)+(a^2)^(1/2))/((a^2)^(1/4)*(a*cot(x))^(1/2)*2^(1/2)-a*co
t(x)-(a^2)^(1/2)))+2*(a^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(a*cot(x))^(1/2)+(
a^2)^(1/4))/(a^2)^(1/4))+2*(a^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(a*cot(x))^(
1/2)-(a^2)^(1/4))/(a^2)^(1/4))-8*(a*cot(x))^(1/2))/cot(x)/(a*cot(x))^(1/2)
```

Maxima [A] time = 1.62105, size = 127, normalized size = 0.72

$$-\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - sqrt(2)*log(sqrt(2)*sqrt(
tan(x)) + tan(x) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1))*sq
rt(a) - 2*sqrt(a)/sqrt(tan(x))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cot^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*cot(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cot(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cot(x)^3), x)
```

3.31 $\int \frac{1}{\sqrt{a\cot^3(x)}}\,dx$

Optimal. Leaf size=176

$$\frac{2\cot(x)}{\sqrt{a\cot^3(x)}}+\frac{\cot^{\frac{3}{2}}(x)\log\left(\cot(x)-\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}\sqrt{a\cot^3(x)}}-\frac{\cot^{\frac{3}{2}}(x)\log\left(\cot(x)+\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}\sqrt{a\cot^3(x)}}-\frac{\cot^{\frac{3}{2}}(x)\tan^{-1}\left(1-\sqrt{2}\right.}{\sqrt{2}\sqrt{a\cot^3(x)}}$$

```
[Out] (2*Cot[x])/Sqrt[a*Cot[x]^3] - (ArcTan[1 - Sqrt[2]*Sqrt[Cot[x]]]*Cot[x]^(3/2
))/(Sqrt[2]*Sqrt[a*Cot[x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Cot[x]]]*Cot[x]^(3
/2))/(Sqrt[2]*Sqrt[a*Cot[x]^3]) + (Cot[x]^(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[x]
] + Cot[x]])/(2*Sqrt[2]*Sqrt[a*Cot[x]^3]) - (Cot[x]^(3/2)*Log[1 + Sqrt[2]*S
qrt[Cot[x]] + Cot[x]])/(2*Sqrt[2]*Sqrt[a*Cot[x]^3])
```

Rubi [A] time = 0.0902691, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {3658, 3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2\cot(x)}{\sqrt{a\cot^3(x)}}+\frac{\cot^{\frac{3}{2}}(x)\log\left(\cot(x)-\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}\sqrt{a\cot^3(x)}}-\frac{\cot^{\frac{3}{2}}(x)\log\left(\cot(x)+\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}\sqrt{a\cot^3(x)}}-\frac{\cot^{\frac{3}{2}}(x)\tan^{-1}\left(1-\sqrt{2}\right.}{\sqrt{2}\sqrt{a\cot^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cot[x]^3],x]
```

```
[Out] (2*Cot[x])/Sqrt[a*Cot[x]^3] - (ArcTan[1 - Sqrt[2]*Sqrt[Cot[x]]]*Cot[x]^(3/2
))/(Sqrt[2]*Sqrt[a*Cot[x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Cot[x]]]*Cot[x]^(3
/2))/(Sqrt[2]*Sqrt[a*Cot[x]^3]) + (Cot[x]^(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[x]
] + Cot[x]])/(2*Sqrt[2]*Sqrt[a*Cot[x]^3]) - (Cot[x]^(3/2)*Log[1 + Sqrt[2]*S
qrt[Cot[x]] + Cot[x]])/(2*Sqrt[2]*Sqrt[a*Cot[x]^3])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a\cot^3(x)}}\,dx &= \frac{\cot^{\frac{3}{2}}(x)\int \frac{1}{\cot^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\cot^3(x)}} \\
&= \frac{2\cot(x)}{\sqrt{a\cot^3(x)}} - \frac{\cot^{\frac{3}{2}}(x)\int \sqrt{\cot(x)}\,dx}{\sqrt{a\cot^3(x)}} \\
&= \frac{2\cot(x)}{\sqrt{a\cot^3(x)}} + \frac{\cot^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \cot(x)\right)}{\sqrt{a\cot^3(x)}} \\
&= \frac{2\cot(x)}{\sqrt{a\cot^3(x)}} + \frac{\left(2\cot^{\frac{3}{2}}(x)\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\cot(x)}\right)}{\sqrt{a\cot^3(x)}} \\
&= \frac{2\cot(x)}{\sqrt{a\cot^3(x)}} - \frac{\cot^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(x)}\right)}{\sqrt{a\cot^3(x)}} + \frac{\cot^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\cot(x)}\right)}{\sqrt{a\cot^3(x)}} \\
&= \frac{2\cot(x)}{\sqrt{a\cot^3(x)}} + \frac{\cot^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\cot(x)}\right)}{2\sqrt{a\cot^3(x)}} + \frac{\cot^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\cot \cdots}\right.}{2\sqrt{a\cot^3(x)}} \cdots \\
&= \frac{2\cot(x)}{\sqrt{a\cot^3(x)}} + \frac{\cot^{\frac{3}{2}}(x)\log\left(1-\sqrt{2}\sqrt{\cot(x)}+\cot(x)\right)}{2\sqrt{2}\sqrt{a\cot^3(x)}} - \frac{\cot^{\frac{3}{2}}(x)\log\left(1+\sqrt{2}\sqrt{\cot(x)}+\cot(x)\right)}{2\sqrt{2}\sqrt{a\cot^3(x)}} + \cdots \\
&= \frac{2\cot(x)}{\sqrt{a\cot^3(x)}} - \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(x)}\right)\cot^{\frac{3}{2}}(x)}{\sqrt{2}\sqrt{a\cot^3(x)}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(x)}\right)\cot^{\frac{3}{2}}(x)}{\sqrt{2}\sqrt{a\cot^3(x)}} + \frac{\cot^{\frac{3}{2}}(x)\log\left(1 \cdots\right.}{2\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.0112449, size = 28, normalized size = 0.16

$$\frac{2\cot(x)\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\cot^2(x)\right)}{\sqrt{a\cot^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cot[x]^3],x]
```

```
[Out] (2*Cot[x]*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[x]^2])/Sqrt[a*Cot[x]^3]
```

Maple [A] time = 0.078, size = 164, normalized size = 0.9

$$\frac{\cot(x)}{4}\left(\sqrt{2}\sqrt{a\cot(x)}\ln\left(-\left(\sqrt[4]{a^2}\sqrt{a\cot(x)}\sqrt{2}-a\cot(x)-\sqrt{a^2}\right)\left(a\cot(x)+\sqrt[4]{a^2}\sqrt{a\cot(x)}\sqrt{2}+\sqrt{a^2}\right)^{-1}\right)+2\sqrt{2}\sqrt{a\cot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)^3)^(1/2),x)
```

```
[Out] 1/4*cot(x)*(2^(1/2)*(a*cot(x))^(1/2)*ln(-((a^2)^(1/4)*(a*cot(x))^(1/2)*2^(1
/2)-a*cot(x)-(a^2)^(1/2))/(a*cot(x)+(a^2)^(1/4)*(a*cot(x))^(1/2)*2^(1/2)+(a
^2)^(1/2)))+2*2^(1/2)*(a*cot(x))^(1/2)*arctan((2^(1/2)*(a*cot(x))^(1/2)+(a^
2)^(1/4))/(a^2)^(1/4))+2*2^(1/2)*(a*cot(x))^(1/2)*arctan((2^(1/2)*(a*cot(x)
)^(1/2)-(a^2)^(1/4))/(a^2)^(1/4))+8*(a^2)^(1/4))/(a*cot(x)^3)^(1/2)/(a^2)^(
1/4)
```

Maxima [A] time = 1.69194, size = 127, normalized size = 0.72

$$-\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) + sqrt(2)*log(sqrt(2)*sqrt(
tan(x)) + tan(x) + 1) - sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1))/sq
rt(a) + 2*sqrt(tan(x))/sqrt(a)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cot^{3}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cot(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cot(x)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cot(x)^3), x)
```

3.32 $\int \frac{1}{\left(a\cot^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{2}{3a\sqrt{a\cot^3(x)}}+\frac{\cot^{\frac{3}{2}}(x)\log\left(\cot(x)-\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}a\sqrt{a\cot^3(x)}}-\frac{\cot^{\frac{3}{2}}(x)\log\left(\cot(x)+\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}a\sqrt{a\cot^3(x)}}+\frac{2\tan^2(x)}{7a\sqrt{a\cot^3(x)}}+$$

```
[Out] -2/(3*a*Sqrt[a*Cot[x]^3]) + (ArcTan[1 - Sqrt[2]*Sqrt[Cot[x]]]*Cot[x]^(3/2))
/(Sqrt[2]*a*Sqrt[a*Cot[x]^3]) - (ArcTan[1 + Sqrt[2]*Sqrt[Cot[x]]]*Cot[x]^(3
/2))/(Sqrt[2]*a*Sqrt[a*Cot[x]^3]) + (Cot[x]^(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[
x]] + Cot[x]])/(2*Sqrt[2]*a*Sqrt[a*Cot[x]^3]) - (Cot[x]^(3/2)*Log[1 + Sqrt[
2]*Sqrt[Cot[x]] + Cot[x]])/(2*Sqrt[2]*a*Sqrt[a*Cot[x]^3]) + (2*Tan[x]^2)/(7
*a*Sqrt[a*Cot[x]^3])
```

Rubi [A] time = 0.096023, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {3658, 3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2}{3a\sqrt{a\cot^3(x)}}+\frac{\cot^{\frac{3}{2}}(x)\log\left(\cot(x)-\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}a\sqrt{a\cot^3(x)}}-\frac{\cot^{\frac{3}{2}}(x)\log\left(\cot(x)+\sqrt{2}\sqrt{\cot(x)}+1\right)}{2\sqrt{2}a\sqrt{a\cot^3(x)}}+\frac{2\tan^2(x)}{7a\sqrt{a\cot^3(x)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x]^3)^(-3/2),x]
```

```
[Out] -2/(3*a*Sqrt[a*Cot[x]^3]) + (ArcTan[1 - Sqrt[2]*Sqrt[Cot[x]]]*Cot[x]^(3/2))
/(Sqrt[2]*a*Sqrt[a*Cot[x]^3]) - (ArcTan[1 + Sqrt[2]*Sqrt[Cot[x]]]*Cot[x]^(3
/2))/(Sqrt[2]*a*Sqrt[a*Cot[x]^3]) + (Cot[x]^(3/2)*Log[1 - Sqrt[2]*Sqrt[Cot[
x]] + Cot[x]])/(2*Sqrt[2]*a*Sqrt[a*Cot[x]^3]) - (Cot[x]^(3/2)*Log[1 + Sqrt[
2]*Sqrt[Cot[x]] + Cot[x]])/(2*Sqrt[2]*a*Sqrt[a*Cot[x]^3]) + (2*Tan[x]^2)/(7
*a*Sqrt[a*Cot[x]^3])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a\cot^3(x)\right)^{3/2}}\,dx = \frac{\cot^{\frac{3}{2}}(x)\int \frac{1}{\cot^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\cot^3(x)}}$$

$$= \frac{2\tan^2(x)}{7a\sqrt{a\cot^3(x)}} - \frac{\cot^{\frac{3}{2}}(x)\int \frac{1}{\cot^{\frac{5}{2}}(x)}\,dx}{a\sqrt{a\cot^3(x)}}$$

$$= -\frac{2}{3a\sqrt{a\cot^3(x)}} + \frac{2\tan^2(x)}{7a\sqrt{a\cot^3(x)}} + \frac{\cot^{\frac{3}{2}}(x)\int \frac{1}{\sqrt{\cot(x)}}\,dx}{a\sqrt{a\cot^3(x)}}$$

$$= -\frac{2}{3a\sqrt{a\cot^3(x)}} + \frac{2\tan^2(x)}{7a\sqrt{a\cot^3(x)}} - \frac{\cot^{\frac{3}{2}}(x)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \cot(x)\right)}{a\sqrt{a\cot^3(x)}}$$

$$= -\frac{2}{3a\sqrt{a\cot^3(x)}} + \frac{2\tan^2(x)}{7a\sqrt{a\cot^3(x)}} - \frac{\left(2\cot^{\frac{3}{2}}(x)\right)\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\cot(x)}\right)}{a\sqrt{a\cot^3(x)}}$$

$$= -\frac{2}{3a\sqrt{a\cot^3(x)}} + \frac{2\tan^2(x)}{7a\sqrt{a\cot^3(x)}} - \frac{\cot^{\frac{3}{2}}(x)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\cot(x)}\right)}{a\sqrt{a\cot^3(x)}} - \frac{\cot^{\frac{3}{2}}(x)\,\mathrm{Subst}\left(\int \frac{1}{1 \ldots}\right.}{a\sqrt{a\,\mathrm{co}\ldots}}$$

$$= -\frac{2}{3a\sqrt{a\cot^3(x)}} + \frac{2\tan^2(x)}{7a\sqrt{a\cot^3(x)}} - \frac{\cot^{\frac{3}{2}}(x)\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\cot(x)}\right)}{2a\sqrt{a\cot^3(x)}} - \frac{\cot^{\frac{3}{2}}(x)\,\mathrm{Subst}\ldots}{2\ldots}$$

$$= -\frac{2}{3a\sqrt{a\cot^3(x)}} + \frac{\cot^{\frac{3}{2}}(x)\log\left(1-\sqrt{2}\sqrt{\cot(x)}+\cot(x)\right)}{2\sqrt{2}a\sqrt{a\cot^3(x)}} - \frac{\cot^{\frac{3}{2}}(x)\log\left(1+\sqrt{2}\sqrt{\cot(x)}+\cot(x\ldots\right.}{2\sqrt{2}a\sqrt{a\cot^3(x)}}$$

$$= -\frac{2}{3a\sqrt{a\cot^3(x)}} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\cot(x)}\right)\cot^{\frac{3}{2}}(x)}{\sqrt{2}a\sqrt{a\cot^3(x)}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\cot(x)}\right)\cot^{\frac{3}{2}}(x)}{\sqrt{2}a\sqrt{a\cot^3(x)}} + \frac{\cot^{\frac{3}{2}}(x)\ldots}{\ldots}$$

Mathematica [C] time = 0.0142202, size = 30, normalized size = 0.14

$$\frac{2\cot(x)\text{Hypergeometric2F1}\left(-\frac{7}{4},1,-\frac{3}{4},-\cot^2(x)\right)}{7\left(a\cot^3(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x]^3)^(-3/2),x]
```

```
[Out] (2*Cot[x]*Hypergeometric2F1[-7/4, 1, -3/4, -Cot[x]^2])/(7*(a*Cot[x]^3)^(3/2
))
```

Maple [A] time = 0.051, size = 185, normalized size = 0.9

$$-\frac{\cot(x)}{84\,a^4}\left(21\,\sqrt[4]{a^2}\sqrt{2}\,(a\cot(x))^{7/2}\ln\left(-\frac{a\cot(x)+\sqrt[4]{a^2}\sqrt{a\cot(x)}\sqrt{2}+\sqrt{a^2}}{\sqrt[4]{a^2}\sqrt{a\cot(x)}\sqrt{2}-a\cot(x)-\sqrt{a^2}}\right)+42\,\sqrt[4]{a^2}\sqrt{2}\,(a\cot(x))^{7/2}\arctan\left(\frac{\sqrt{2}\sqrt{a\cot}}{\sqrt[4]{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)^3)^(3/2),x)
```

```
[Out] -1/84*cot(x)/a^4*(21*(a^2)^(1/4)*2^(1/2)*(a*cot(x))^(7/2)*ln(-(a*cot(x)+(a^
2)^(1/4)*(a*cot(x))^(1/2)*2^(1/2)+(a^2)^(1/2))/((a^2)^(1/4)*(a*cot(x))^(1/2
)*2^(1/2)-a*cot(x)-(a^2)^(1/2)))+42*(a^2)^(1/4)*2^(1/2)*(a*cot(x))^(7/2)*ar
ctan((2^(1/2)*(a*cot(x))^(1/2)+(a^2)^(1/4))/(a^2)^(1/4))+42*(a^2)^(1/4)*2^(
1/2)*(a*cot(x))^(7/2)*arctan((2^(1/2)*(a*cot(x))^(1/2)-(a^2)^(1/4))/(a^2)^(
1/4))+56*cot(x)^2*a^4-24*a^4)/(a*cot(x)^3)^(3/2)
```

Maxima [A] time = 1.57804, size = 147, normalized size = 0.69

$$\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)+}{4\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - sqrt(2)*log(sqrt(2)*sqrt(t
an(x)) + tan(x) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1))/a^(
3/2) + 2/21*(3*sqrt(a)*tan(x)^(7/2) - 7*sqrt(a)*tan(x)^(3/2))/a^2
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cot^{3}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)**3)**(3/2),x)
```

```
[Out] Integral((a*cot(x)**3)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cot\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cot(x)^3)^(-3/2), x)
```

3.33 $\int \left(a \cot^4(x)\right)^{3/2} \, dx$

Optimal. Leaf size=70

$$-\frac{1}{5}a\cot^3(x)\sqrt{a\cot^4(x)}+\frac{1}{3}a\cot(x)\sqrt{a\cot^4(x)}-ax\tan^2(x)\sqrt{a\cot^4(x)}-a\tan(x)\sqrt{a\cot^4(x)}$$

```
[Out] (a*Cot[x]*Sqrt[a*Cot[x]^4])/3 - (a*Cot[x]^3*Sqrt[a*Cot[x]^4])/5 - a*Sqrt[a*
Cot[x]^4]*Tan[x] - a*x*Sqrt[a*Cot[x]^4]*Tan[x]^2
```

Rubi [A] time = 0.0271455, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3658, 3473, 8}

$$-\frac{1}{5}a\cot^3(x)\sqrt{a\cot^4(x)}+\frac{1}{3}a\cot(x)\sqrt{a\cot^4(x)}-ax\tan^2(x)\sqrt{a\cot^4(x)}-a\tan(x)\sqrt{a\cot^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x]^4)^(3/2),x]
```

```
[Out] (a*Cot[x]*Sqrt[a*Cot[x]^4])/3 - (a*Cot[x]^3*Sqrt[a*Cot[x]^4])/5 - a*Sqrt[a*
Cot[x]^4]*Tan[x] - a*x*Sqrt[a*Cot[x]^4]*Tan[x]^2
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a\cot^4(x)\right)^{3/2}\,dx &= \left(a\sqrt{a\cot^4(x)}\tan^2(x)\right)\int \cot^6(x)\,dx\\
&= -\frac{1}{5}a\cot^3(x)\sqrt{a\cot^4(x)}-\left(a\sqrt{a\cot^4(x)}\tan^2(x)\right)\int \cot^4(x)\,dx\\
&= \frac{1}{3}a\cot(x)\sqrt{a\cot^4(x)}-\frac{1}{5}a\cot^3(x)\sqrt{a\cot^4(x)}+\left(a\sqrt{a\cot^4(x)}\tan^2(x)\right)\int \cot^2(x)\,dx\\
&= \frac{1}{3}a\cot(x)\sqrt{a\cot^4(x)}-\frac{1}{5}a\cot^3(x)\sqrt{a\cot^4(x)}-a\sqrt{a\cot^4(x)}\tan(x)-\left(a\sqrt{a\cot^4(x)}\tan^2(x)\right)\int 1\\
&= \frac{1}{3}a\cot(x)\sqrt{a\cot^4(x)}-\frac{1}{5}a\cot^3(x)\sqrt{a\cot^4(x)}-a\sqrt{a\cot^4(x)}\tan(x)-ax\sqrt{a\cot^4(x)}\tan^2(x)
\end{aligned}$$

Mathematica [A] time = 0.135134, size = 39, normalized size = 0.56

$$-\frac{1}{15}\tan^6(x)\left(a\cot^4(x)\right)^{3/2}\left(15x+\cot(x)\left(3\csc^4(x)-11\csc^2(x)+23\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x]^4)^(3/2),x]
```

```
[Out] -((a*Cot[x]^4)^(3/2)*(15*x + Cot[x]*(23 - 11*Csc[x]^2 + 3*Csc[x]^4))*Tan[x]
^6)/15
```

Maple [A] time = 0.06, size = 40, normalized size = 0.6

$$\frac{1}{15\left(\cot(x)\right)^6}\left(a\left(\cot(x)\right)^4\right)^{\frac{3}{2}}\left(-3\left(\cot(x)\right)^5+5\left(\cot(x)\right)^3+\frac{15\pi}{2}-15\,\mathrm{arccot}\left(\cot(x)\right)-15\cot(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)^4)^(3/2),x)
```

```
[Out] 1/15*(a*cot(x)^4)^(3/2)*(-3*cot(x)^5+5*cot(x)^3+15/2*Pi-15*arccot(cot(x))-1
5*cot(x))/cot(x)^6
```

Maxima [A] time = 1.65532, size = 50, normalized size = 0.71

$$-a^{\frac{3}{2}}x - \frac{15\,a^{\frac{3}{2}}\tan(x)^4 - 5\,a^{\frac{3}{2}}\tan(x)^2 + 3\,a^{\frac{3}{2}}}{15\,\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -a^(3/2)*x - 1/15*(15*a^(3/2)*tan(x)^4 - 5*a^(3/2)*tan(x)^2 + 3*a^(3/2))/ta
n(x)^5
```

Fricas [A] time = 2.17736, size = 289, normalized size = 4.13

$$\frac{\left(23\,a\cos(2\,x)^3 - a\cos(2\,x)^2 - 11\,a\cos(2\,x) + 15\left(ax\cos(2\,x)^2 - 2\,ax\cos(2\,x) + ax\right)\sin(2\,x) + 13\,a\right)\sqrt{\frac{a\cos(2\,x)^2+2\,a\cos(2\,x)}{\cos(2\,x)^2-2\,\cos(2\,x)}}}{15\left(\cos(2\,x)^2 - 1\right)\sin(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(23*a*cos(2*x)^3 - a*cos(2*x)^2 - 11*a*cos(2*x) + 15*(a*x*cos(2*x)^2 -
 2*a*x*cos(2*x) + a*x)*sin(2*x) + 13*a)*sqrt((a*cos(2*x)^2 + 2*a*cos(2*x) +
 a)/(cos(2*x)^2 - 2*cos(2*x) + 1))/((cos(2*x)^2 - 1)*sin(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot^4(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)**4)**(3/2),x)
```

```
[Out] Integral((a*cot(x)**4)**(3/2), x)
```

Giac [A] time = 1.23163, size = 77, normalized size = 1.1

$$\frac{1}{480}\left(3\tan\left(\frac{1}{2}x\right)^{5}-35\tan\left(\frac{1}{2}x\right)^{3}-480x-\frac{330\tan\left(\frac{1}{2}x\right)^{4}-35\tan\left(\frac{1}{2}x\right)^{2}+3}{\tan\left(\frac{1}{2}x\right)^{5}}+330\tan\left(\frac{1}{2}x\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/480*(3*tan(1/2*x)^5 - 35*tan(1/2*x)^3 - 480*x - (330*tan(1/2*x)^4 - 35*ta
n(1/2*x)^2 + 3)/tan(1/2*x)^5 + 330*tan(1/2*x))*a^(3/2)
```

3.34 $\int \sqrt{a\cot^4(x)}\,dx$

Optimal. Leaf size=32

$$-x\tan^2(x)\sqrt{a\cot^4(x)}-\tan(x)\sqrt{a\cot^4(x)}$$

```
[Out] -(Sqrt[a*Cot[x]^4]*Tan[x]) - x*Sqrt[a*Cot[x]^4]*Tan[x]^2
```

Rubi [A] time = 0.0154425, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3658, 3473, 8}

$$-x\tan^2(x)\sqrt{a\cot^4(x)}-\tan(x)\sqrt{a\cot^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cot[x]^4],x]
```

```
[Out] -(Sqrt[a*Cot[x]^4]*Tan[x]) - x*Sqrt[a*Cot[x]^4]*Tan[x]^2
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a \cot^4(x)}\,dx &= \left(\sqrt{a \cot^4(x)}\tan^2(x)\right)\int \cot^2(x)\,dx\\ &= -\sqrt{a\cot^4(x)}\tan(x) - \left(\sqrt{a\cot^4(x)}\tan^2(x)\right)\int 1\,dx\\ &= -\sqrt{a\cot^4(x)}\tan(x) - x\sqrt{a\cot^4(x)}\tan^2(x)\end{aligned}$$

Mathematica [A] time = 0.0153746, size = 20, normalized size = 0.62

$$\tan^2(x)(x+\cot(x))\left(-\sqrt{a\cot^4(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cot[x]^4],x]
```

```
[Out] -(Sqrt[a*Cot[x]^4]*(x + Cot[x])*Tan[x]^2)
```

Maple [A] time = 0.078, size = 27, normalized size = 0.8

$$\frac{1}{(\cot(x))^2}\sqrt{a(\cot(x))^4}\left(-\cot(x)+\frac{\pi}{2}-\operatorname{arccot}(\cot(x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)^4)^(1/2),x)
```

```
[Out] (a*cot(x)^4)^(1/2)/cot(x)^2*(-cot(x)+1/2*Pi-arccot(cot(x)))
```

Maxima [A] time = 1.64365, size = 22, normalized size = 0.69

$$-\sqrt{a}x - \frac{\sqrt{a}}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*x - sqrt(a)/tan(x)
```

Fricas [B] time = 2.15884, size = 154, normalized size = 4.81

$$\frac{(x\cos(2x) - x - \sin(2x))\sqrt{\frac{a\cos(2x)^2 + 2a\cos(2x) + a}{\cos(2x)^2 - 2\cos(2x) + 1}}}{\cos(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*cos(2*x) - x - sin(2*x))*sqrt((a*cos(2*x)^2 + 2*a*cos(2*x) + a)/(cos(2*x
)^2 - 2*cos(2*x) + 1))/(cos(2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cot^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*cot(x)**4), x)
```

Giac [A] time = 1.29934, size = 28, normalized size = 0.88

$$-\frac{1}{2}\sqrt{a}\left(2x + \frac{1}{\tan\left(\frac{1}{2}x\right)} - \tan\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(2*x + 1/tan(1/2*x) - tan(1/2*x))
```

3.35 $\int \frac{1}{\sqrt{a\cot^4(x)}}\,dx$

Optimal. Leaf size=31

$$\frac{\cot(x)}{\sqrt{a\cot^4(x)}} - \frac{x\cot^2(x)}{\sqrt{a\cot^4(x)}}$$

```
[Out] Cot[x]/Sqrt[a*Cot[x]^4] - (x*Cot[x]^2)/Sqrt[a*Cot[x]^4]
```

Rubi [A] time = 0.0161043, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3658, 3473, 8}

$$\frac{\cot(x)}{\sqrt{a\cot^4(x)}} - \frac{x\cot^2(x)}{\sqrt{a\cot^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cot[x]^4],x]
```

```
[Out] Cot[x]/Sqrt[a*Cot[x]^4] - (x*Cot[x]^2)/Sqrt[a*Cot[x]^4]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\cot^4(x)}}\,dx &= \frac{\cot^2(x)\int \tan^2(x)\,dx}{\sqrt{a\cot^4(x)}}\\ &= \frac{\cot(x)}{\sqrt{a\cot^4(x)}} - \frac{\cot^2(x)\int 1\,dx}{\sqrt{a\cot^4(x)}}\\ &= \frac{\cot(x)}{\sqrt{a\cot^4(x)}} - \frac{x\cot^2(x)}{\sqrt{a\cot^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.0224911, size = 21, normalized size = 0.68

$$\frac{\cot(x) - x\cot^2(x)}{\sqrt{a\cot^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cot[x]^4],x]
```

```
[Out] (Cot[x] - x*Cot[x]^2)/Sqrt[a*Cot[x]^4]
```

Maple [A] time = 0.084, size = 26, normalized size = 0.8

$$\cot(x)\left(\left(\frac{\pi}{2} - \operatorname{arccot}(\cot(x))\right)\cot(x) + 1\right)\frac{1}{\sqrt{a(\cot(x))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)^4)^(1/2),x)
```

```
[Out] cot(x)*((1/2*Pi-arccot(cot(x)))*cot(x)+1)/(a*cot(x)^4)^(1/2)
```

Maxima [A] time = 1.52214, size = 18, normalized size = 0.58

$$-\frac{x}{\sqrt{a}}+\frac{\tan(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -x/sqrt(a) + tan(x)/sqrt(a)
```

Fricas [B] time = 2.10897, size = 203, normalized size = 6.55

$$\frac{\left(x\cos(2x)^2-(\cos(2x)-1)\sin(2x)-x\right)\sqrt{\frac{a\cos(2x)^2+2a\cos(2x)+a}{\cos(2x)^2-2\cos(2x)+1}}}{a\cos(2x)^2+2a\cos(2x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*cos(2*x)^2 - (cos(2*x) - 1)*sin(2*x) - x)*sqrt((a*cos(2*x)^2 + 2*a*cos(2
*x) + a)/(cos(2*x)^2 - 2*cos(2*x) + 1))/(a*cos(2*x)^2 + 2*a*cos(2*x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\cot^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cot(x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.36 $\int \frac{1}{\left(a\cot^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{x\cot^2(x)}{a\sqrt{a\cot^4(x)}}+\frac{\cot(x)}{a\sqrt{a\cot^4(x)}}+\frac{\tan^3(x)}{5a\sqrt{a\cot^4(x)}}-\frac{\tan(x)}{3a\sqrt{a\cot^4(x)}}$$

```
[Out] Cot[x]/(a*Sqrt[a*Cot[x]^4]) - (x*Cot[x]^2)/(a*Sqrt[a*Cot[x]^4]) - Tan[x]/(3
*a*Sqrt[a*Cot[x]^4]) + Tan[x]^3/(5*a*Sqrt[a*Cot[x]^4])
```

Rubi [A] time = 0.0257301, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3658, 3473, 8}

$$-\frac{x\cot^2(x)}{a\sqrt{a\cot^4(x)}}+\frac{\cot(x)}{a\sqrt{a\cot^4(x)}}+\frac{\tan^3(x)}{5a\sqrt{a\cot^4(x)}}-\frac{\tan(x)}{3a\sqrt{a\cot^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x]^4)^(-3/2),x]
```

```
[Out] Cot[x]/(a*Sqrt[a*Cot[x]^4]) - (x*Cot[x]^2)/(a*Sqrt[a*Cot[x]^4]) - Tan[x]/(3
*a*Sqrt[a*Cot[x]^4]) + Tan[x]^3/(5*a*Sqrt[a*Cot[x]^4])
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\cot^4(x)\right)^{3/2}}\,dx &= \frac{\cot^2(x)\int \tan^6(x)\,dx}{a\sqrt{a\cot^4(x)}}\\
&= \frac{\tan^3(x)}{5a\sqrt{a\cot^4(x)}} - \frac{\cot^2(x)\int \tan^4(x)\,dx}{a\sqrt{a\cot^4(x)}}\\
&= -\frac{\tan(x)}{3a\sqrt{a\cot^4(x)}} + \frac{\tan^3(x)}{5a\sqrt{a\cot^4(x)}} + \frac{\cot^2(x)\int \tan^2(x)\,dx}{a\sqrt{a\cot^4(x)}}\\
&= \frac{\cot(x)}{a\sqrt{a\cot^4(x)}} - \frac{\tan(x)}{3a\sqrt{a\cot^4(x)}} + \frac{\tan^3(x)}{5a\sqrt{a\cot^4(x)}} - \frac{\cot^2(x)\int 1\,dx}{a\sqrt{a\cot^4(x)}}\\
&= \frac{\cot(x)}{a\sqrt{a\cot^4(x)}} - \frac{x\cot^2(x)}{a\sqrt{a\cot^4(x)}} - \frac{\tan(x)}{3a\sqrt{a\cot^4(x)}} + \frac{\tan^3(x)}{5a\sqrt{a\cot^4(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.10534, size = 42, normalized size = 0.55

$$\frac{-15x\cot^2(x) + 23\cot(x) + \csc(x)\sec(x)\left(3\sec^2(x) - 11\right)}{15a\sqrt{a\cot^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x]^4)^(-3/2),x]
```

```
[Out] (23*Cot[x] - 15*x*Cot[x]^2 + Csc[x]*Sec[x]*(-11 + 3*Sec[x]^2))/(15*a*Sqrt[a
*Cot[x]^4])
```

Maple [A] time = 0.047, size = 42, normalized size = 0.6

$$\frac{\cot(x)}{15}\left(15\,(\pi/2 - \mathrm{arccot}\,(\cot(x)))\,(\cot(x))^5 + 15\,(\cot(x))^4 - 5\,(\cot(x))^2 + 3\right)\left(a\,(\cot(x))^4\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)^4)^(3/2),x)
```

```
[Out] 1/15*cot(x)*(15*(1/2*Pi-arccot(cot(x)))*cot(x)^5+15*cot(x)^4-5*cot(x)^2+3)/
(a*cot(x)^4)^(3/2)
```

Maxima [A] time = 1.66123, size = 39, normalized size = 0.51

$$\frac{3\,\tan\left(x\right)^{5}-5\,\tan\left(x\right)^{3}+15\,\tan\left(x\right)}{15\,a^{\frac{3}{2}}}-\frac{x}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(x)^5 - 5*tan(x)^3 + 15*tan(x))/a^(3/2) - x/a^(3/2)
```

Fricas [B] time = 1.62312, size = 367, normalized size = 4.77

$$\frac{\left(15\,x\cos\left(2\,x\right)^{4}+30\,x\cos\left(2\,x\right)^{3}-30\,x\cos\left(2\,x\right)-\left(23\,\cos\left(2\,x\right)^{3}+\cos\left(2\,x\right)^{2}-11\,\cos\left(2\,x\right)-13\right)\sin\left(2\,x\right)-15\,x\right)\sqrt{\frac{a\cos(2}{\cos(2}}}{15\left(a^{2}\cos\left(2\,x\right)^{4}+4\,a^{2}\cos\left(2\,x\right)^{3}+6\,a^{2}\cos\left(2\,x\right)^{2}+4\,a^{2}\cos\left(2\,x\right)+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*x*cos(2*x)^4 + 30*x*cos(2*x)^3 - 30*x*cos(2*x) - (23*cos(2*x)^3 +
cos(2*x)^2 - 11*cos(2*x) - 13)*sin(2*x) - 15*x)*sqrt((a*cos(2*x)^2 + 2*a*co
s(2*x) + a)/(cos(2*x)^2 - 2*cos(2*x) + 1))/(a^2*cos(2*x)^4 + 4*a^2*cos(2*x)
^3 + 6*a^2*cos(2*x)^2 + 4*a^2*cos(2*x) + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cot^{4}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)**4)**(3/2),x)
```

```
[Out] Integral((a*cot(x)**4)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.37 $\int (b\cot^p(c+dx))^n\,dx$

Optimal. Leaf size=60

$$-\frac{\cot(c+dx)\,(b\cot^p(c+dx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\cot^2(c+dx)\right)}{d(np+1)}$$

```
[Out] -((Cot[c + d*x]*(b*Cot[c + d*x]^p)^n*Hypergeometric2F1[1, (1 + n*p)/2, (3 +
 n*p)/2, -Cot[c + d*x]^2])/(d*(1 + n*p)))
```

Rubi [A] time = 0.0397252, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3659, 3476, 364}

$$-\frac{\cot(c+dx)\,(b\cot^p(c+dx))^n\;{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\cot^2(c+dx)\right)}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cot[c + d*x]^p)^n,x]
```

```
[Out] -((Cot[c + d*x]*(b*Cot[c + d*x]^p)^n*Hypergeometric2F1[1, (1 + n*p)/2, (3 +
 n*p)/2, -Cot[c + d*x]^2])/(d*(1 + n*p)))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (b\cot^p(c+dx))^n\,dx &= \left(\cot^{-np}(c+dx)\,(b\cot^p(c+dx))^n\right)\int \cot^{np}(c+dx)\,dx\\ &= -\frac{\left(\cot^{-np}(c+dx)\,(b\cot^p(c+dx))^n\right)\mathrm{Subst}\left(\int \frac{x^{np}}{1+x^2}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\cot(c+dx)\,(b\cot^p(c+dx))^n\;{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\cot^2(c+dx)\right)}{d(1+np)}\end{aligned}$$

Mathematica [A] time = 0.0516629, size = 58, normalized size = 0.97

$$-\frac{\cot(c+dx)\,(b\cot^p(c+dx))^n\,\mathrm{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\cot^2(c+dx)\right)}{dnp+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cot[c + d*x]^p)^n,x]
```

```
[Out] -((Cot[c + d*x]*(b*Cot[c + d*x]^p)^n*Hypergeometric2F1[1, (1 + n*p)/2, (3 +
 n*p)/2, -Cot[c + d*x]^2])/(d + d*n*p))
```

Maple [F] time = 4.602, size = 0, normalized size = 0.

$$\int \left(b\left(\cot\left(dx+c\right)\right)^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cot(d*x+c)^p)^n,x)
```

```
[Out] int((b*cot(d*x+c)^p)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cot{(dx+c)}^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(d*x+c)^p)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cot(d*x + c)^p)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cot{(dx+c)}^p\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(d*x+c)^p)^n,x, algorithm="fricas")
```

```
[Out] integral((b*cot(d*x + c)^p)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cot^{p}{(c+dx)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(d*x+c)**p)**n,x)
```

```
[Out] Integral((b*cot(c + d*x)**p)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cot{(dx+c)}^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(d*x+c)^p)^n,x, algorithm="giac")
```

```
[Out] integrate((b*cot(d*x + c)^p)^n, x)
```

3.38 $\int (a(b\cot(c+dx))^p)^n \, dx$

Optimal. Leaf size=62

$$-\frac{\cot(c+dx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\cot^2(c+dx)\right)(a(b\cot(c+dx))^p)^n}{d(np+1)}$$

```
[Out] -((Cot[c + d*x]*(a*(b*Cot[c + d*x])^p)^n*Hypergeometric2F1[1, (1 + n*p)/2,
(3 + n*p)/2, -Cot[c + d*x]^2])/(d*(1 + n*p)))
```

Rubi [A] time = 0.0449681, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3659, 3476, 364}

$$-\frac{\cot(c+dx)\, {}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\cot^2(c+dx)\right)(a(b\cot(c+dx))^p)^n}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*Cot[c + d*x])^p)^n,x]
```

```
[Out] -((Cot[c + d*x]*(a*(b*Cot[c + d*x])^p)^n*Hypergeometric2F1[1, (1 + n*p)/2,
(3 + n*p)/2, -Cot[c + d*x]^2])/(d*(1 + n*p)))
```

Rule 3659

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Tan[e + f*x])^n)^FracPart[p])/(c*Tan[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a(b\cot(c+dx))^p)^n\,dx &= \left((b\cot(c+dx))^{-np}\,(a(b\cot(c+dx))^p)^n\right)\int (b\cot(c+dx))^{np}\,dx\\ &= -\frac{\left(b(b\cot(c+dx))^{-np}\,(a(b\cot(c+dx))^p)^n\right)\text{Subst}\left(\int \frac{x^{np}}{b^2+x^2}\,dx, x, b\cot(c+dx)\right)}{d}\\ &= -\frac{\cot(c+dx)\,(a(b\cot(c+dx))^p)^n\;{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\cot^2(c+dx)\right)}{d(1+np)}\end{aligned}$$

Mathematica [A] time = 0.0473146, size = 60, normalized size = 0.97

$$-\frac{\cot(c+dx)\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\cot^2(c+dx)\right)(a(b\cot(c+dx))^p)^n}{dnp+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*Cot[c + d*x])^p)^n,x]
```

```
[Out] -((Cot[c + d*x]*(a*(b*Cot[c + d*x])^p)^n*Hypergeometric2F1[1, (1 + n*p)/2,
(3 + n*p)/2, -Cot[c + d*x]^2])/(d + d*n*p))
```

Maple [F] time = 5.854, size = 0, normalized size = 0.

$$\int \left(a\,(b\cot(dx+c))^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*cot(d*x+c))^p)^n,x)
```

```
[Out] int((a*(b*cot(d*x+c))^p)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b\cot (dx + c))^p a\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cot(d*x+c))^p)^n,x, algorithm="maxima")
```

```
[Out] integrate(((b*cot(d*x + c))^p*a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left((b\cot (dx + c))^p a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cot(d*x+c))^p)^n,x, algorithm="fricas")
```

```
[Out] integral(((b*cot(d*x + c))^p*a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a (b\cot (c + dx))^p\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cot(d*x+c))**p)**n,x)
```

```
[Out] Integral((a*(b*cot(c + d*x))**p)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b\cot (dx + c))^p a\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cot(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*cot(d*x + c))^p*a)^n, x)
```

3.39 $\int (b\cot(e+fx))^n (a\sin(e+fx))^m\,dx$

Optimal. Leaf size=87

$$-\frac{(a\sin(e+fx))^m(b\cot(e+fx))^{n+1}\sin^2(e+fx)^{\frac{1}{2}(-m+n+1)}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1),\frac{n+3}{2},\cos^2(e+fx)\right)}{bf(n+1)}$$

```
[Out] -(((b*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[(1 + n)/2, (1 - m + n)/2, (3
+ n)/2, Cos[e + f*x]^2]*(a*Sin[e + f*x])^m*(Sin[e + f*x]^2)^((1 - m + n)/2)
)/(b*f*(1 + n)))
```

Rubi [A] time = 0.0966597, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2603, 2617}

$$-\frac{(a\sin(e+fx))^m(b\cot(e+fx))^{n+1}\sin^2(e+fx)^{\frac{1}{2}(-m+n+1)}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1);\frac{n+3}{2};\cos^2(e+fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cot[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] -(((b*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[(1 + n)/2, (1 - m + n)/2, (3
+ n)/2, Cos[e + f*x]^2]*(a*Sin[e + f*x])^m*(Sin[e + f*x]^2)^((1 - m + n)/2)
)/(b*f*(1 + n)))
```

Rule 2603

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int (b\cot(e+fx))^n(a\sin(e+fx))^m\,dx = \left(\left(\frac{\csc(e+fx)}{a}\right)^m (a\sin(e+fx))^m\right)\int (b\cot(e+fx))^n\left(\frac{\csc(e+fx)}{a}\right)^{-m}\,dx$$
$$= -\frac{(b\cot(e+fx))^{1+n}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1-m+n);\frac{3+n}{2};\cos^2(e+fx)\right)(a\sin(e+fx))^m}{bf(1+n)}$$

Mathematica [C] time = 1.72175, size = 289, normalized size = 3.32

$$\frac{(m-n+3)\sin(e+fx)(a\sin(e+}{f(m-n+1)\left((m-n+3)F_1\left(\frac{1}{2}(m-n+1);-n,m+1;\frac{1}{2}(m-n+3);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\tan^2\left(\frac{1}{2}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Cot[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] ((3 + m - n)*AppellF1[(1 + m - n)/2, -n, 1 + m, (3 + m - n)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*(b*Cot[e + f*x])^n*Sin[e + f*x]*(a*Sin[e + f*x
])^m)/(f*(1 + m - n)*((3 + m - n)*AppellF1[(1 + m - n)/2, -n, 1 + m, (3 + m
 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(n*AppellF1[(3 + m -
n)/2, 1 - n, 1 + m, (5 + m - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (1 + m)*AppellF1[(3 + m - n)/2, -n, 2 + m, (5 + m - n)/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

Maple [F] time = 1.175, size = 0, normalized size = 0.

$$\int \left(b\cot\left(fx+e\right)\right)^n\left(a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cot(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

```
[Out] int((b*cot(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cot(fx+e))^n (a\sin(fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cot(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cot(fx+e))^n (a\sin(fx+e))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*cot(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))**n*(a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cot(fx+e))^n (a\sin(fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

3.40 $\int (a\cos(e+fx))^m (b\cot(e+fx))^n\,dx$

Optimal. Leaf size=84

$$-\frac{\sin^2(e+fx)^{\frac{n+1}{2}}(a\cos(e+fx))^m(b\cot(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\cos^2(e+fx)\right)}{bf(m+n+1)}$$

```
[Out] -(((a*Cos[e + f*x])^m*(b*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[(1 + n)/2,
 (1 + m + n)/2, (3 + m + n)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((1 + n)/2)
)/(b*f*(1 + m + n)))
```

Rubi [A] time = 0.101382, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2602, 2576}

$$-\frac{\sin^2(e+fx)^{\frac{n+1}{2}}(a\cos(e+fx))^m(b\cot(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(e+fx)\right)}{bf(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Cot[e + f*x])^n,x]
```

```
[Out] -(((a*Cos[e + f*x])^m*(b*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[(1 + n)/2,
 (1 + m + n)/2, (3 + m + n)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((1 + n)/2)
)/(b*f*(1 + m + n)))
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (a\cos(e+fx))^m(b\cot(e+fx))^n\,dx = -\frac{\left(a(a\cos(e+fx))^{-1-n}(b\cot(e+fx))^{1+n}(-\sin(e+fx))^{1+n}\right)\int (a\cos(e+fx))^{m}}{b}$$

$$= -\frac{(a\cos(e+fx))^m(b\cot(e+fx))^{1+n}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\cos^{2}\right.}{bf(1+m+n)}$$

Mathematica [A] time = 0.510996, size = 83, normalized size = 0.99

$$-\frac{b\sec^2(e+fx)^{m/2}(a\cos(e+fx))^m(b\cot(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{m+2}{2},\frac{1-n}{2},\frac{3-n}{2},-\tan^2(e+fx)\right)}{f(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Cot[e + f*x])^n,x]
```

```
[Out] -((b*(a*Cos[e + f*x])^m*(b*Cot[e + f*x])^(-1 + n)*Hypergeometric2F1[(2 + m)
/2, (1 - n)/2, (3 - n)/2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2))/(f*(-1 +
 n)))
```

Maple [F] time = 1.02, size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^m\left(b\cot\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*cot(f*x+e))^n,x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*cot(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(fx+e\right)\right)^m\left(b\cot\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*cot(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*cot(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a\cos\left(fx+e\right)\right)^{m}\left(b\cot\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*cot(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(f*x + e))^m*(b*cot(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*cot(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(fx+e\right)\right)^{m}\left(b\cot\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*cot(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*cot(f*x + e))^n, x)
```

3.41 $\int (a\cot(e+fx))^m (b\cot(e+fx))^n \, dx$

Optimal. Leaf size=64

$$-\frac{(a\cot(e+fx))^{m+1}(b\cot(e+fx))^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),-\cot^2(e+fx)\right)}{af(m+n+1)}$$

```
[Out] -(((a*Cot[e + f*x])^(1 + m)*(b*Cot[e + f*x])^n*Hypergeometric2F1[1, (1 + m
+ n)/2, (3 + m + n)/2, -Cot[e + f*x]^2])/(a*f*(1 + m + n)))
```

Rubi [A] time = 0.0373172, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3476, 364}

$$-\frac{(a\cot(e+fx))^{m+1}(b\cot(e+fx))^n \, _2F_1\left(1,\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);-\cot^2(e+fx)\right)}{af(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[e + f*x])^m*(b*Cot[e + f*x])^n,x]
```

```
[Out] -(((a*Cot[e + f*x])^(1 + m)*(b*Cot[e + f*x])^n*Hypergeometric2F1[1, (1 + m
+ n)/2, (3 + m + n)/2, -Cot[e + f*x]^2])/(a*f*(1 + m + n)))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (a\cot(e+fx))^m (b\cot(e+fx))^n\,dx = \left((a\cot(e+fx))^{-n}(b\cot(e+fx))^n\right)\int (a\cot(e+fx))^{m+n}\,dx$$
$$= -\frac{\left(a(a\cot(e+fx))^{-n}(b\cot(e+fx))^n\right)\text{Subst}\left(\int \frac{x^{m+n}}{a^2+x^2}\,dx, x, a\cot(e+fx)\right)}{f}$$
$$= -\frac{(a\cot(e+fx))^{1+m}(b\cot(e+fx))^n\,{}_2F_1\left(1,\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);-\cot^2(e$$
$$af(1+m+n)$$

Mathematica [A] time = 0.0879458, size = 67, normalized size = 1.05

$$-\frac{\cot(e+fx)(a\cot(e+fx))^m(b\cot(e+fx))^n\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+1)+1,-\cot^2(e+fx)\right)}{f(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[e + f*x])^m*(b*Cot[e + f*x])^n,x]
```

```
[Out] -((Cot[e + f*x]*(a*Cot[e + f*x])^m*(b*Cot[e + f*x])^n*Hypergeometric2F1[1,
(1 + m + n)/2, 1 + (1 + m + n)/2, -Cot[e + f*x]^2])/(f*(1 + m + n)))
```

Maple [F] time = 0.507, size = 0, normalized size = 0.

$$\int \left(a\cot\left(fx+e\right)\right)^m \left(b\cot\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(f*x+e))^m*(b*cot(f*x+e))^n,x)
```

```
[Out] int((a*cot(f*x+e))^m*(b*cot(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot\left(fx+e\right)\right)^m \left(b\cot\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*(b*cot(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cot(f*x + e))^m*(b*cot(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\cot\left(fx+e\right)\right)^m \left(b\cot\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*(b*cot(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cot(f*x + e))^m*(b*cot(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot\left(e+fx\right)\right)^m \left(b\cot\left(e+fx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))**m*(b*cot(f*x+e))**n,x)
```

```
[Out] Integral((a*cot(e + f*x))**m*(b*cot(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cot\left(fx+e\right)\right)^m \left(b\cot\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*(b*cot(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cot(f*x + e))^m*(b*cot(f*x + e))^n, x)
```

3.42 $\int (b\cot(e+fx))^n (a\sec(e+fx))^m \, dx$

Optimal. Leaf size=90

$$-\frac{\sin^2(e+fx)^{\frac{n+1}{2}}(a\sec(e+fx))^m(b\cot(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),\cos^2(e\right.}{bf(-m+n+1)}$$

```
[Out] -(((b*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[(1 + n)/2, (1 - m + n)/2, (3
- m + n)/2, Cos[e + f*x]^2]*(a*Sec[e + f*x])^m*(Sin[e + f*x]^2)^((1 + n)/2)
)/(b*f*(1 - m + n)))
```

Rubi [A] time = 0.153449, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2618, 2602, 2576}

$$-\frac{\sin^2(e+fx)^{\frac{n+1}{2}}(a\sec(e+fx))^m(b\cot(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);\cos^2(e+fx)\right)}{bf(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cot[e + f*x])^n*(a*Sec[e + f*x])^m,x]
```



```
[Out] -(((b*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[(1 + n)/2, (1 - m + n)/2, (3
- m + n)/2, Cos[e + f*x]^2]*(a*Sec[e + f*x])^m*(Sin[e + f*x]^2)^((1 + n)/2)
)/(b*f*(1 - m + n)))
```


Rule 2618

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (b\cot(e+fx))^n(a\sec(e+fx))^m\,dx = \left(\left(\frac{\cos(e+fx)}{a}\right)^m(a\sec(e+fx))^m\right)\int\left(\frac{\cos(e+fx)}{a}\right)^{-m}(b\cot(e+fx))^n\,dx$$

$$= -\frac{\left(\left(\frac{\cos(e+fx)}{a}\right)^{-1+m-n}(b\cot(e+fx))^{1+n}(a\sec(e+fx))^m(-\sin(e+fx))^{1+n}\right)\int\left(\frac{\cos\ldots}{\ldots}\right.}{ab}$$

$$= -\frac{(b\cot(e+fx))^{1+n}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);\cos^2(e+fx)\right)(a\sec(e\ldots}{bf(1-m+n)}$$

Mathematica [A] time = 0.44545, size = 83, normalized size = 0.92

$$-\frac{b\sec^2(e+fx)^{-m/2}(a\sec(e+fx))^m(b\cot(e+fx))^{n-1}\text{Hypergeometric2F1}\left(1-\frac{m}{2},\frac{1-n}{2},\frac{3-n}{2},-\tan^2(e+fx)\right)}{f(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cot[e + f*x])^n*(a*Sec[e + f*x])^m,x]
```

```
[Out] -((b*(b*Cot[e + f*x])^(-1 + n)*Hypergeometric2F1[1 - m/2, (1 - n)/2, (3 - n
)/2, -Tan[e + f*x]^2]*(a*Sec[e + f*x])^m)/(f*(-1 + n)*(Sec[e + f*x]^2)^(m/2
)))
```

Maple [F] time = 1.068, size = 0, normalized size = 0.

$$\int\left(b\cot\left(fx+e\right)\right)^n\left(a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cot(f*x+e))^n*(a*sec(f*x+e))^m,x)
```

```
[Out] int((b*cot(f*x+e))^n*(a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cot\left(fx+e\right)\right)^{n}\left(a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cot(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cot\left(fx+e\right)\right)^{n}\left(a\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*cot(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(e+fx\right)\right)^{m}\left(b\cot\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))**n*(a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*sec(e + f*x))**m*(b*cot(e + f*x))**n, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cot\left(fx+e\right)\right)^{n}\left(a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

3.43 $\int (d\cot(e+fx))^n \csc^6(e+fx)\,dx$

Optimal. Leaf size=76

$$-\frac{2(d\cot(e+fx))^{n+3}}{d^3f(n+3)}-\frac{(d\cot(e+fx))^{n+5}}{d^5f(n+5)}-\frac{(d\cot(e+fx))^{n+1}}{df(n+1)}$$

```
[Out] -((d*Cot[e + f*x])^(1 + n)/(d*f*(1 + n))) - (2*(d*Cot[e + f*x])^(3 + n))/(d
^3*f*(3 + n)) - (d*Cot[e + f*x])^(5 + n)/(d^5*f*(5 + n))
```

Rubi [A] time = 0.070749, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 270}

$$-\frac{2(d\cot(e+fx))^{n+3}}{d^3f(n+3)}-\frac{(d\cot(e+fx))^{n+5}}{d^5f(n+5)}-\frac{(d\cot(e+fx))^{n+1}}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*Csc[e + f*x]^6,x]
```

```
[Out] -((d*Cot[e + f*x])^(1 + n)/(d*f*(1 + n))) - (2*(d*Cot[e + f*x])^(3 + n))/(d
^3*f*(3 + n)) - (d*Cot[e + f*x])^(5 + n)/(d^5*f*(5 + n))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (d\cot(e+fx))^n \csc^6(e+fx)\,dx = \frac{\text{Subst}\left(\int(-dx)^n\left(1+x^2\right)^2\,dx, x, -\cot(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int\left((-dx)^n+\frac{2(-dx)^{2+n}}{d^2}+\frac{(-dx)^{4+n}}{d^4}\right)dx, x, -\cot(e+fx)\right)}{f}$$
$$= -\frac{(d\cot(e+fx))^{1+n}}{df(1+n)} - \frac{2(d\cot(e+fx))^{3+n}}{d^3f(3+n)} - \frac{(d\cot(e+fx))^{5+n}}{d^5f(5+n)}$$

Mathematica [A] time = 0.233779, size = 73, normalized size = 0.96

$$-\frac{\cot(e+fx)\csc^4(e+fx)\left(-2(n+3)\cos(2(e+fx))+\cos(4(e+fx))+n^2+6n+8\right)(d\cot(e+fx))^n}{f(n+1)(n+3)(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^n*Csc[e + f*x]^6,x]
```

```
[Out] -(((8 + 6*n + n^2 - 2*(3 + n)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*Cot[e +
f*x]*(d*Cot[e + f*x])^n*Csc[e + f*x]^4)/(f*(1 + n)*(3 + n)*(5 + n)))
```

Maple [C] time = 2.195, size = 21900, normalized size = 288.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*csc(f*x+e)^6,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75107, size = 358, normalized size = 4.71

$$-\frac{\left(8\cos\left(fx+e\right)^{5}-4\,(n+5)\cos\left(fx+e\right)^{3}+\left(n^{2}+8\,n+15\right)\cos\left(fx+e\right)\right)\left(\frac{d\cos(fx+e)}{\sin(fx+e)}\right)^{n}}{\left(\left(fn^{3}+9\,fn^{2}+23\,fn+15\,f\right)\cos\left(fx+e\right)^{4}+fn^{3}+9\,fn^{2}-2\left(fn^{3}+9\,fn^{2}+23\,fn+15\,f\right)\cos\left(fx+e\right)^{2}+23\,fn+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -(8*cos(f*x + e)^5 - 4*(n + 5)*cos(f*x + e)^3 + (n^2 + 8*n + 15)*cos(f*x +
e))*(d*cos(f*x + e)/sin(f*x + e))^n/(((f*n^3 + 9*f*n^2 + 23*f*n + 15*f)*cos
(f*x + e)^4 + f*n^3 + 9*f*n^2 - 2*(f*n^3 + 9*f*n^2 + 23*f*n + 15*f)*cos(f*x
 + e)^2 + 23*f*n + 15*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*csc(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cot\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n*csc(f*x + e)^6, x)
```

3.44 $\int (d \cot (e+fx))^n \csc ^4(e+fx) \, dx$

Optimal. Leaf size=51

$$-\frac{(d\cot(e+fx))^{n+3}}{d^3 f(n+3)}-\frac{(d\cot(e+fx))^{n+1}}{df(n+1)}$$

```
[Out] -((d*Cot[e + f*x])^(1 + n)/(d*f*(1 + n))) - (d*Cot[e + f*x])^(3 + n)/(d^3*f
*(3 + n))
```

Rubi [A] time = 0.0527326, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 14}

$$-\frac{(d\cot(e+fx))^{n+3}}{d^3 f(n+3)}-\frac{(d\cot(e+fx))^{n+1}}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*Csc[e + f*x]^4,x]
```

```
[Out] -((d*Cot[e + f*x])^(1 + n)/(d*f*(1 + n))) - (d*Cot[e + f*x])^(3 + n)/(d^3*f
*(3 + n))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int (d\cot(e+fx))^n \csc^4(e+fx)\,dx = \frac{\text{Subst}\left(\int (-dx)^n\left(1+x^2\right)dx, x, -\cot(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((-dx)^n + \frac{(-dx)^{2+n}}{d^2}\right)dx, x, -\cot(e+fx)\right)}{f}$$
$$= -\frac{(d\cot(e+fx))^{1+n}}{df(1+n)} - \frac{(d\cot(e+fx))^{3+n}}{d^3f(3+n)}$$

Mathematica [A] time = 0.127219, size = 45, normalized size = 0.88

$$-\frac{\cot(e+fx)\left((n+1)\csc^2(e+fx)+2\right)(d\cot(e+fx))^n}{f(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^n*Csc[e + f*x]^4,x]
```

```
[Out] -((Cot[e + f*x]*(d*Cot[e + f*x])^n*(2 + (1 + n)*Csc[e + f*x]^2))/(f*(1 + n)
*(3 + n)))
```

Maple [C] time = 0.757, size = 10907, normalized size = 213.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*csc(f*x+e)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73169, size = 204, normalized size = 4.

$$\frac{\left(2\,\cos\left(fx+e\right)^{3}-(n+3)\cos\left(fx+e\right)\right)\left(\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)^{n}}{\left(fn^{2}-\left(fn^{2}+4\,fn+3\,f\right)\cos\left(fx+e\right)^{2}+4\,fn+3\,f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] (2*cos(f*x + e)^3 - (n + 3)*cos(f*x + e))*(d*cos(f*x + e)/sin(f*x + e))^n/(
(f*n^2 - (f*n^2 + 4*f*n + 3*f)*cos(f*x + e)^2 + 4*f*n + 3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*csc(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cot\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n*csc(f*x + e)^4, x)
```

3.45 $\int (d\cot(e+fx))^n \csc^2(e+fx)\,dx$

Optimal. Leaf size=25

$$-\frac{(d\cot(e+fx))^{n+1}}{df(n+1)}$$

```
[Out] -((d*Cot[e + f*x])^(1 + n)/(d*f*(1 + n)))
```

Rubi [A] time = 0.042452, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 32}

$$-\frac{(d\cot(e+fx))^{n+1}}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*Csc[e + f*x]^2,x]
```

```
[Out] -((d*Cot[e + f*x])^(1 + n)/(d*f*(1 + n)))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d\cot(e+fx))^n \csc^2(e+fx)\,dx &= \frac{\text{Subst}\left(\int (-dx)^n\,dx, x, -\cot(e+fx)\right)}{f}\\ &= -\frac{(d\cot(e+fx))^{1+n}}{df(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0193327, size = 26, normalized size = 1.04

$$-\frac{\cot(e+fx)(d\cot(e+fx))^n}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^n*Csc[e + f*x]^2,x]
```

```
[Out] -((Cot[e + f*x]*(d*Cot[e + f*x])^n)/(f*(1 + n)))
```

Maple [A] time = 0.03, size = 26, normalized size = 1.

$$-\frac{\left(d\cot\left(fx+e\right)\right)^{1+n}}{fd\left(1+n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*csc(f*x+e)^2,x)
```

```
[Out] -(d*cot(f*x+e))^(1+n)/d/f/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72804, size = 97, normalized size = 3.88

$$-\frac{\left(\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)^{n}\cos\left(fx+e\right)}{\left(fn+f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -(d*cos(f*x + e)/sin(f*x + e))^n*cos(f*x + e)/((f*n + f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(e+fx\right)\right)^{n}\csc^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*csc(f*x+e)**2,x)
```

```
[Out] Integral((d*cot(e + f*x))**n*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n*csc(f*x + e)^2, x)
```

3.46 $\int (d\cot(e+fx))^n \sin^2(e+fx)\,dx$

Optimal. Leaf size=51

$$-\frac{(d\cot(e+fx))^{n+1}\text{Hypergeometric2F1}\left(2,\frac{n+1}{2},\frac{n+3}{2},-\cot^2(e+fx)\right)}{df(n+1)}$$

```
[Out] -(((d*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Cot
[e + f*x]^2])/(d*f*(1 + n)))
```

Rubi [A] time = 0.0491075, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 364}

$$-\frac{(d\cot(e+fx))^{n+1}\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*Sin[e + f*x]^2,x]
```


```
[Out] -(((d*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Cot
[e + f*x]^2])/(d*f*(1 + n)))
```


Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (d\cot(e+fx))^n\sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-dx)^n}{(1+x^2)^2}\,dx, x, -\cot(e+fx)\right)}{f}$$

$$= -\frac{(d\cot(e+fx))^{1+n}\,{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};-\cot^2(e+fx)\right)}{df(1+n)}$$

Mathematica [C] time = 3.07399, size = 509, normalized size = 9.98

$$-\frac{}{f(n-1)\left(2(n-3)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(\frac{1}{2}-\frac{n}{2};-n,2;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2(n-3)\cos^2\left(\frac{1}{2}(e+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cot[e + f*x])^n*Sin[e + f*x]^2,x]
```

```
[Out] (-4*(-3 + n)*(AppellF1[1/2 - n/2, -n, 2, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] - AppellF1[1/2 - n/2, -n, 3, 3/2 - n/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^3*(d*Cot[e + f*x])^n*Sin[(e + f*x
)/2]*Sin[e + f*x]^2)/(f*(-1 + n)*(2*(-3 + n)*AppellF1[1/2 - n/2, -n, 2, 3/2
 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 2*(-3
 + n)*AppellF1[1/2 - n/2, -n, 3, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Cos[(e + f*x)/2]^2 - 2*(n*AppellF1[3/2 - n/2, 1 - n, 2, 5/2 - n/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[3/2 - n/2, 1 - n, 3
, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*AppellF1[3/2 - n/
2, -n, 3, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 3*AppellF1[
3/2 - n/2, -n, 4, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1
+ Cos[e + f*x])))
```

Maple [F] time = 1.099, size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^n\left(\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*sin(f*x+e)^2,x)
```

```
[Out] int((d*cot(f*x+e))^n*sin(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cot (fx+e))^n \sin (fx+e)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cot(f*x + e))^n*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos (fx+e)^2-1\right)(d\cot (fx+e))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(d*cot(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\cot (e+fx))^n \sin^2 (e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*sin(f*x+e)**2,x)
```

```
[Out] Integral((d*cot(e + f*x))**n*sin(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^n \sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n*sin(f*x + e)^2, x)
```

3.47 $\int (d\cot(e+fx))^n \sin^4(e+fx)\,dx$

Optimal. Leaf size=51

$$-\frac{(d\cot(e+fx))^{n+1}\text{Hypergeometric2F1}\left(3,\frac{n+1}{2},\frac{n+3}{2},-\cot^2(e+fx)\right)}{df(n+1)}$$

```
[Out] -(((d*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Cot
[e + f*x]^2])/(d*f*(1 + n)))
```

Rubi [A] time = 0.0479054, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2607, 364}

$$-\frac{(d\cot(e+fx))^{n+1}\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};-\cot^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*Sin[e + f*x]^4,x]
```

```
[Out] -(((d*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Cot
[e + f*x]^2])/(d*f*(1 + n)))
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (d\cot(e+fx))^n \sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-dx)^n}{(1+x^2)^3}\,dx, x, -\cot(e+fx)\right)}{f}$$
$$= -\frac{(d\cot(e+fx))^{1+n}\,{}_2F_1\left(3, \frac{1+n}{2}; \frac{3+n}{2}; -\cot^2(e+fx)\right)}{df(1+n)}$$

Mathematica [C] time = 7.33255, size = 1099, normalized size = 21.55

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cot[e + f*x])^n*Sin[e + f*x]^4,x]
```

```
[Out] (2*(-3 + n)*(AppellF1[1/2 - n/2, -n, 3, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] - 2*AppellF1[1/2 - n/2, -n, 4, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + AppellF1[1/2 - n/2, -n, 5, 3/2 - n/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^3*(d*Cot[e + f*x])^n*Sin[(
e + f*x)/2]*Sin[e + f*x]^4)/(f*(-1 + n)*(-3*AppellF1[3/2 - n/2, -n, 4, 5/2
- n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 8*AppellF1[3/2 - n/2, -n,
 5, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 5*AppellF1[3/2 -
n/2, -n, 6, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 3*AppellF
1[1/2 - n/2, -n, 3, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos
[(e + f*x)/2]^2 - n*AppellF1[1/2 - n/2, -n, 3, 3/2 - n/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 6*AppellF1[1/2 - n/2, -n, 4, 3
/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*n
*AppellF1[1/2 - n/2, -n, 4, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[(e + f*x)/2]^2 + 3*AppellF1[1/2 - n/2, -n, 5, 3/2 - n/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 - n*AppellF1[1/2 - n/2,
-n, 5, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]
^2 + n*AppellF1[3/2 - n/2, 1 - n, 3, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(-1 + Cos[e + f*x]) + n*AppellF1[3/2 - n/2, 1 - n, 5, 5/2 - n
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(-1 + Cos[e + f*x]) + 3*Appell
F1[3/2 - n/2, -n, 4, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Co
s[e + f*x] - 8*AppellF1[3/2 - n/2, -n, 5, 5/2 - n/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Cos[e + f*x] + 5*AppellF1[3/2 - n/2, -n, 6, 5/2 - n/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 4*n*AppellF1[3/2 - n
/2, 1 - n, 4, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e +
f*x)/2]^2))
```

Maple [F] time = 1.15, size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{n} \left(\sin\left(fx+e\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*sin(f*x+e)^4,x)
```

```
[Out] int((d*cot(f*x+e))^n*sin(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{n} \sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cot(f*x + e))^n*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(fx+e\right)^{4} - 2\,\cos\left(fx+e\right)^{2} + 1\right)\left(d\cot\left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(d*cot(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n*sin(f*x + e)^4, x)
```

3.48 $\int (d\cot(e+fx))^n \csc^3(e+fx)\,dx$

Optimal. Leaf size=79

$$-\frac{\csc^3(e+fx)\sin^2(e+fx)^{\frac{n+4}{2}}(d\cot(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+4}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{df(n+1)}$$

```
[Out] -(((d*Cot[e + f*x])^(1 + n)*Csc[e + f*x]^3*Hypergeometric2F1[(1 + n)/2, (4
+ n)/2, (3 + n)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((4 + n)/2))/(d*f*(1 +
n)))
```

Rubi [A] time = 0.0405785, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$-\frac{\csc^3(e+fx)\sin^2(e+fx)^{\frac{n+4}{2}}(d\cot(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+4}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*Csc[e + f*x]^3,x]
```

```
[Out] -(((d*Cot[e + f*x])^(1 + n)*Csc[e + f*x]^3*Hypergeometric2F1[(1 + n)/2, (4
+ n)/2, (3 + n)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((4 + n)/2))/(d*f*(1 +
n)))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\cot(e+fx))^n\csc^3(e+fx)\,dx = -\frac{(d\cot(e+fx))^{1+n}\csc^3(e+fx)\,{}_2F_1\left(\frac{1+n}{2},\frac{4+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{4+n}{2}}}{df(1+n)}$$

Mathematica [B] time = 6.51261, size = 190, normalized size = 2.41

$$-\frac{\tan^2\left(\frac{1}{2}(e+fx)\right)(d\cot(e+fx))^n\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^{-n}\left((n-2)n\cot^4\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cot[e + f*x])^n*Csc[e + f*x]^3,x]
```

```
[Out] -((d*Cot[e + f*x])^n*((-2 + n)*n*Cot[(e + f*x)/2]^4*Hypergeometric2F1[-1 -
n/2, -n, -n/2, Tan[(e + f*x)/2]^2] + (2 + n)*(n*Hypergeometric2F1[1 - n/2,
-n, 2 - n/2, Tan[(e + f*x)/2]^2] + 2*(-2 + n)*Cot[(e + f*x)/2]^2*Hypergeome
tric2F1[-n, -n/2, 1 - n/2, Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2]^2)/(4*f*n
*(-4 + n^2)*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n)
```

Maple [F] time = 0.551, size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{n}\left(\csc\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*csc(f*x+e)^3,x)
```

```
[Out] int((d*cot(f*x+e))^n*csc(f*x+e)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*cot(f*x + e))^n*csc(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\cot\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((d*cot(f*x + e))^n*csc(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cot\left(e+fx\right)\right)^{n}\csc^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*csc(f*x+e)**3,x)
```

```
[Out] Integral((d*cot(e + f*x))**n*csc(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cot\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n*csc(f*x + e)^3, x)
```

3.49 $\int (d\cot(e+fx))^n \csc(e+fx)\,dx$

Optimal. Leaf size=77

$$-\frac{\csc(e+fx)\sin^2(e+fx)^{\frac{n+2}{2}}(d\cot(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{df(n+1)}$$

```
[Out] -(((d*Cot[e + f*x])^(1 + n)*Csc[e + f*x]*Hypergeometric2F1[(1 + n)/2, (2 +
n)/2, (3 + n)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((2 + n)/2))/(d*f*(1 + n)
))
```

Rubi [A] time = 0.029108, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2617}

$$-\frac{\csc(e+fx)\sin^2(e+fx)^{\frac{n+2}{2}}(d\cot(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*Csc[e + f*x],x]
```



```
[Out] -(((d*Cot[e + f*x])^(1 + n)*Csc[e + f*x]*Hypergeometric2F1[(1 + n)/2, (2 +
n)/2, (3 + n)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((2 + n)/2))/(d*f*(1 + n)
))
```


Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\cot(e+fx))^n\csc(e+fx)\,dx = -\frac{(d\cot(e+fx))^{1+n}\csc(e+fx)\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{2+n}{2}}}{df(1+n)}$$

Mathematica [A] time = 0.13063, size = 69, normalized size = 0.9

$$-\frac{(d\cot(e+fx))^n\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^{-n}\text{Hypergeometric2F1}\left(-n,-\frac{n}{2},1-\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^n*Csc[e + f*x],x]
```

```
[Out] -(((d*Cot[e + f*x])^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, Tan[(e + f*x)/2]
^2])/(f*n*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n))
```

Maple [F] time = 0.499, size = 0, normalized size = 0.

$$\int\left(d\cot\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*csc(f*x+e),x)
```

```
[Out] int((d*cot(f*x+e))^n*csc(f*x+e),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cot\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((d*cot(f*x + e))^n*csc(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\cot\left(fx+e\right)\right)^{n}\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((d*cot(f*x + e))^n*csc(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(e+fx\right)\right)^{n}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*csc(f*x+e),x)
```

```
[Out] Integral((d*cot(e + f*x))**n*csc(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*csc(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n*csc(f*x + e), x)
```

3.50 $\int (d\cot(e+fx))^n \sin(e+fx)\,dx$

Optimal. Leaf size=73

$$-\frac{\sin(e+fx)\sin^2(e+fx)^{n/2}(d\cot(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{df(n+1)}$$

```
[Out] -(((d*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[n/2, (1 + n)/2, (3 + n)/2, Co
s[e + f*x]^2]*Sin[e + f*x]*(Sin[e + f*x]^2)^(n/2))/(d*f*(1 + n)))
```

Rubi [A] time = 0.0411376, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2617}

$$-\frac{\sin(e+fx)\sin^2(e+fx)^{n/2}(d\cot(e+fx))^{n+1}\,{}_2F_1\left(\frac{n}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*Sin[e + f*x],x]
```

```
[Out] -(((d*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[n/2, (1 + n)/2, (3 + n)/2, Co
s[e + f*x]^2]*Sin[e + f*x]*(Sin[e + f*x]^2)^(n/2))/(d*f*(1 + n)))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\cot(e+fx))^n \sin(e+fx)\,dx = -\frac{(d\cot(e+fx))^{1+n}\,{}_2F_1\left(\frac{n}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\sin(e+fx)\sin^2(e+fx)^{n/2}}{df(1+n)}$$

Mathematica [C] time = 1.05132, size = 264, normalized size = 3.62

$$-\frac{8(n-4)\sin^2\left(\frac{1}{2}(e+fx)\right)\cos^4\left(\frac{1}{2}(e+fx)\right)F_1\left(1-\frac{n}{2};-\right.}{f(n-2)\left(2(n-4)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(1-\frac{n}{2};-n,2;2-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2(\cos(e+fx)-1)\left(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cot[e + f*x])^n*Sin[e + f*x],x]
```

```
[Out] (-8*(-4 + n)*AppellF1[1 - n/2, -n, 2, 2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[(e + f*x)/2]^4*(d*Cot[e + f*x])^n*Sin[(e + f*x)/2]^2)/(f*(
-2 + n)*(2*(-4 + n)*AppellF1[1 - n/2, -n, 2, 2 - n/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 2*(n*AppellF1[2 - n/2, 1 - n, 2, 3
 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*AppellF1[2 - n/2, -n,
3, 3 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

Maple [F] time = 1., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^n \sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*sin(f*x+e),x)
```

```
[Out] int((d*cot(f*x+e))^n*sin(f*x+e),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^n \sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((d*cot(f*x + e))^n*sin(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\cot\left(fx+e\right)\right)^{n}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((d*cot(f*x + e))^n*sin(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cot\left(e+fx\right)\right)^{n}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*sin(f*x+e),x)
```

```
[Out] Integral((d*cot(e + f*x))**n*sin(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cot\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n*sin(f*x + e), x)
```

3.51 $\int (d\cot(e+fx))^n \sin^3(e+fx)\,dx$

Optimal. Leaf size=79

$$-\frac{\sin^3(e+fx)\sin^2(e+fx)^{\frac{n-2}{2}}(d\cot(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n-2}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{df(n+1)}$$

```
[Out] -((((d*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[(-2 + n)/2, (1 + n)/2, (3 + n
)/2, Cos[e + f*x]^2]*Sin[e + f*x]^3*(Sin[e + f*x]^2)^((-2 + n)/2))/(d*f*(1
+ n)))
```

Rubi [A] time = 0.0424696, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2617}

$$-\frac{\sin^3(e+fx)\sin^2(e+fx)^{\frac{n-2}{2}}(d\cot(e+fx))^{n+1}{}_2F_1\left(\frac{n-2}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^n*Sin[e + f*x]^3,x]
```

```
[Out] -((((d*Cot[e + f*x])^(1 + n)*Hypergeometric2F1[(-2 + n)/2, (1 + n)/2, (3 + n
)/2, Cos[e + f*x]^2]*Sin[e + f*x]^3*(Sin[e + f*x]^2)^((-2 + n)/2))/(d*f*(1
+ n)))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\cot(e+fx))^n\sin^3(e+fx)\,dx = -\frac{(d\cot(e+fx))^{1+n}\,{}_2F_1\left(\frac{1}{2}(-2+n),\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\sin^3(e+fx)\sin^2(e+fx}{df(1+n)}$$

Mathematica [C] time = 2.30768, size = 477, normalized size = 6.04

$$-\frac{}{f(n-2)\left(2(n-4)\cos^2\left(\frac{1}{2}(e+fx)\right)F_1\left(1-\frac{n}{2};-n,3;2-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2(n-4)\cos^2\left(\frac{1}{2}(e\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cot[e + f*x])^n*Sin[e + f*x]^3,x]
```

```
[Out] (-4*(-4 + n)*(AppellF1[1 - n/2, -n, 3, 2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] - AppellF1[1 - n/2, -n, 4, 2 - n/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*Cos[(e + f*x)/2]^3*(d*Cot[e + f*x])^n*Sin[(e + f*x)/2]*Sin
[e + f*x]^3)/(f*(-2 + n)*(2*(-4 + n)*AppellF1[1 - n/2, -n, 3, 2 - n/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 - 2*(-4 + n)*Appell
F1[1 - n/2, -n, 4, 2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e
 + f*x)/2]^2 - 2*(n*AppellF1[2 - n/2, 1 - n, 3, 3 - n/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - n*AppellF1[2 - n/2, 1 - n, 4, 3 - n/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + 3*AppellF1[2 - n/2, -n, 4, 3 - n/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[2 - n/2, -n, 5, 3 - n/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

Maple [F] time = 1.074, size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{n}\left(\sin\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^n*sin(f*x+e)^3,x)
```

```
[Out] int((d*cot(f*x+e))^n*sin(f*x+e)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*cot(f*x + e))^n*sin(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(d\cot\left(fx+e\right)\right)^{n}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(d*cot(f*x + e))^n*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**n*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cot\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^n*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^n*sin(f*x + e)^3, x)
```

3.52 $\int (b\cot(e+fx))^n (a\csc(e+fx))^m \, dx$

Optimal. Leaf size=83

$$-\frac{(a\csc(e+fx))^m (b\cot(e+fx))^{n+1} \sin^2(e+fx)^{\frac{1}{2}(m+n+1)} \text{Hypergeometric2F1}\left(\frac{n+1}{2}, \frac{1}{2}(m+n+1), \frac{n+3}{2}, \cos^2(e+fx)\right)}{bf(n+1)}$$

```
[Out] -(((b*Cot[e + f*x])^(1 + n)*(a*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n)/2,
 (1 + m + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((1 + m + n)/2)
)/(b*f*(1 + n)))
```

Rubi [A] time = 0.0457312, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2617}

$$-\frac{(a\csc(e+fx))^m (b\cot(e+fx))^{n+1} \sin^2(e+fx)^{\frac{1}{2}(m+n+1)} \, {}_2F_1\left(\frac{n+1}{2}, \frac{1}{2}(m+n+1); \frac{n+3}{2}; \cos^2(e+fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cot[e + f*x])^n*(a*Csc[e + f*x])^m,x]
```

```
[Out] -(((b*Cot[e + f*x])^(1 + n)*(a*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n)/2,
 (1 + m + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((1 + m + n)/2)
)/(b*f*(1 + n)))
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\cot(e+fx))^n (a\csc(e+fx))^m \, dx = -\frac{(b\cot(e+fx))^{1+n} (a\csc(e+fx))^m \, {}_2F_1\left(\frac{1+n}{2}, \frac{1}{2}(1+m+n); \frac{3+n}{2}; \cos^2(e+fx)\right)}{bf(1+n)}$$

Mathematica [C] time = 1.79764, size = 306, normalized size = 3.69

$$-\frac{a(m+n-3)(a\csc(e+fx)}{f(m+n-1)\left(2\tan^2\left(\frac{1}{2}(e+fx)\right)\left(nF_1\left(\frac{1}{2}(-m-n+3);1-n,1-m;\frac{1}{2}(-m-n+5);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Cot[e + f*x])^n*(a*Csc[e + f*x])^m,x]
```

```
[Out] -((a*(-3 + m + n)*AppellF1[(1 - m - n)/2, -n, 1 - m, (3 - m - n)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(b*Cot[e + f*x])^n*(a*Csc[e + f*x])^(-1 +
 m))/(f*(-1 + m + n)*((-3 + m + n)*AppellF1[(1 - m - n)/2, -n, 1 - m, (3 -
m - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*(n*AppellF1[(3 - m -
 n)/2, 1 - n, 1 - m, (5 - m - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] - (-1 + m)*AppellF1[(3 - m - n)/2, -n, 2 - m, (5 - m - n)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 1.115, size = 0, normalized size = 0.

$$\int \left(b\cot\left(fx+e\right)\right)^{n}\left(a\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cot(f*x+e))^n*(a*csc(f*x+e))^m,x)
```

```
[Out] int((b*cot(f*x+e))^n*(a*csc(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cot\left(fx+e\right)\right)^{n}\left(a\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))^n*(a*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cot(f*x + e))^n*(a*csc(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cot\left(fx+e\right)\right)^{n}\left(a\csc\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))^n*(a*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*cot(f*x + e))^n*(a*csc(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\csc\left(e+fx\right)\right)^{m}\left(b\cot\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))**n*(a*csc(f*x+e))**m,x)
```

```
[Out] Integral((a*csc(e + f*x))**m*(b*cot(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\cot\left(fx+e\right)\right)^{n}\left(a\csc\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cot(f*x+e))^n*(a*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*cot(f*x + e))^n*(a*csc(f*x + e))^m, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```